Nonlinear Speed Observer for the PM Stepper Motor

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Abstract—This note considers a nonlinear observer for the PM Stepper Motor. Specifically, the method of Krener and Respondek is used to construct a nontrivial full-order speed observer which has linear error dynamics. The procedure is validated with experimental results. Finally, reduced-order observers are constructed by inspection.

I. INTRODUCTION

Permanent magnet stepper motors are electromechanical incremental motion devices used in positioning applications. These motors were designed to provide precise positioning control to within an integer number of steps without using sensors. They are in fact open-loop stable to any step position and thus, no feedback is needed to control them. However, their step response have quite a bit of overshoot and a relatively long settling time. On the other hand, as these motors are electronically commutated and have no windings on the rotor, they do not have two major disadvantages of DC brush motors: 1) mechanical wear of the commutator and 2) limitation to a 50% duty cycle to allow the armature windings to cool. For these reasons, many positioning systems now use stepper motors with the addition of encoder (position) feedback. Control strategies are then needed to have the performance of the stepper motor approach that of DC brush motors. The feedback linearization approach has been proposed in [1] with modifications for output feedback in [3], [4]. A variable structure approach has been given in [2]. Several of these approaches require state feedback. In this work, a nonlinear speed observer based on measurements of the position and the phase currents is proposed. This observer algorithm is based on the work of Krener and Respondek [9] in which an output transformation is used to obtain a nonlinear observer with linear error dynamics. This approach has been refined in [10], [11]. Some simplification to these algorithms have been recently reported by Phelps [12]. Other work in this area is given in [13]-[15]. As this note shows, the PM stepper motor presents a nice illustration of the multi-output nonlinear observer theory especially in that both the output transformation and state-space transformation are nontrivial. Some preliminary results of this work were presented in a conference proceedings [18].

II. MATHEMATICAL MODELS OF THE PM STEPPER MOTOR

A. Phase A-B Model

The dynamical equations describing the PM stepper motor are [5], [6], [7], [8]

\[
\begin{align*}
\frac{d\theta}{dt} & = \omega \\
\frac{d\omega}{dt} & = \frac{[v_a - Ri_a + K_m \omega \sin (N, \theta)]}{L} \\
\frac{d\theta}{dt} & = \frac{[v_b - Ri_b - K_m \omega \cos (N, \theta)]}{L} \\
\frac{d\omega}{dt} & = \frac{[-K_m i_a \sin (N, \theta) + K_m i_b \cos (N, \theta) - B \omega]}{J} \\
\frac{d\theta}{dt} & = \omega
\end{align*}
\]

\(i_a, i_b, v_a, v_b, \omega, \) are the currents and voltages in phases A and B, respectively. \(L\) and \(R\) are the self-inductance and resistance of each phase winding, \(K_m\) is the motor torque constant, \(N\) is the number of rotor teeth, \(\theta\) is the rotor inertia, \(B\) is the viscous friction constant, \(\omega\) is the rotor speed and \(\theta\) is the motor position. This model neglects the slight magnetic coupling between phases and the small change in \(L\) as a function of position \(\theta\) due to the rotor teeth [6].

With \(K_1 = R/L, K_2 = K_m/L, K_3 = K_m/J, K_4 = B/J\) and \(x_1 = i_a, x_2 = i_b, x_3 = \omega, x_4 = \theta, u_1 = v_a/L, u_2 = v_b/L, u_3 = \omega/J, u_4 = \theta/J\), we may rewrite the above as

\[
\begin{align*}
\frac{dx_1}{dt} & = -K_1 x_1 + K_2 x_2 \sin (N, x_4) + u_1 \\
\frac{dx_2}{dt} & = -K_1 x_2 - K_2 x_3 \cos (N, x_4) + u_2 \\
\frac{dx_3}{dt} & = -K_3 x_3 \sin (N, x_4) + K_3 x_2 \cos (N, x_4) - K_4 x_3 \\
\frac{dx_4}{dt} & = x_3.
\end{align*}
\]

B. Direct-Quadrature Model of the PM Stepper Motor

The direct-quadrature (DQ) transformation is defined by

\[
\begin{bmatrix}
i_d \\
i_q \\
\omega \\
\theta
\end{bmatrix} =
\begin{bmatrix}
cos(N, \theta) & \sin(N, \theta) & 0 & 0 \\
-\sin(N, \theta) & \cos(N, \theta) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
\omega \\
\theta
\end{bmatrix}
\]

Define the DQ voltages by

\[
\begin{bmatrix}
v_d \\
v_q \end{bmatrix} =
\begin{bmatrix}
cos(N, \theta) & \sin(N, \theta) \\
-\sin(N, \theta) & \cos(N, \theta)
\end{bmatrix}
\begin{bmatrix}
v_a \\
v_b
\end{bmatrix}
\]

In terms of these new inputs and state variables, the system description is

\[
\begin{align*}
\frac{d}{dt} (v_d - Ri_d + N, \omega Li_q) & = -L_i_3 + K_m \omega \\
\frac{d}{dt} (v_q - Ri_q - N, \omega Li_q - K_m \omega) & = 0 \\
\frac{d}{dt} \left( K_m i_q - B \omega \right) & = 0 \\
\frac{d}{dt} \theta & = \omega
\end{align*}
\]

As pointed out in [11], the nonlinearity in (5) may be cancelled out by state feedback.

III. NONLINEAR OBSERVERS

A. Speed Observer with Linear Error Dynamics

A nonlinear speed observer is now constructed following the methodology of Krener and Respondek [9] (see also [10], [11]). It would seem quite natural to take as the outputs for the system (2) \(y_1 = x_1, y_2 = x_2\) and \(y_3 = x_4\). However, it turns out that in order to construct an observer with linear error dynamics, it is necessary to do a nonlinear output transformation on \(y_1, y_2\) and \(y_3\). This is a direct consequence of [9, theorems 4.1 and 4.2] as

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will be shown in the next subsection. Define new outputs of (2) as
\[
\begin{bmatrix}
y_1^* \\
y_2^* \\
y_3^*
\end{bmatrix} =
\begin{bmatrix}
h_1(x) \\
h_2(x) \\
h_3(x)
\end{bmatrix} =
\begin{bmatrix}
x_1 + (K_2/N_1) \cos(N_1x_4) \\
x_2 + (K_2/N_1) \sin(N_1x_4) \\
x_4
\end{bmatrix}
\] (6)
and consider a nonlinear state-space transformation defined by
\[
x_1^* = x_1 + (K_2/N_1) \cos(N_1x_4)
\]
\[
x_2^* = x_2 + (K_2/N_1) \sin(N_1x_4)
\]
\[
x_3^* = x_3 + K_4x_4
\]
\[
x_4^* = x_4.
\] (7)
In these coordinates, the system equations become
\[
\dot{x}_1^* = -K_1x_1^* + (K_1K_2/N_1) \cos(N_1x_4^*) + u_1
\]
\[
\dot{x}_2^* = -K_1x_2^* + (K_1K_2/N_1) \sin(N_1x_4^*) + u_2
\]
\[
\dot{x}_3^* = -K_3x_3^* \sin(N_1x_1^*) + K_3x_2^* \cos(N_1x_1^*)
\]
\[
\dot{x}_4^* = x_3^* - K_4x_4^*.
\] (8)
Note that the unmeasured state variable \(x_3^* = \omega + K_4\theta\) appears linearly in (8). With this in mind, (8) may be rewritten as
\[
\dot{x}^* = Ax^* + \varphi(y, u)
\]
y = Cx*
(9)
where
\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
-1 & -1 & -1 & -1
\end{bmatrix},
\]
\[
\varphi(y, u) = \begin{bmatrix}
-K_1y_1 + (K_1K_2/N_1) \cos(N_1y_3) + u_1 \\
-K_1y_2 + (K_1K_2/N_1) \sin(N_1y_3) + u_2 \\
-K_3y_3 \sin(N_1y_1) + K_3y_2 \cos(N_1y_1) \\
-K_4y_3
\end{bmatrix},
\]
\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}.
\]
The observer is then defined by
\[
\dot{x}^* = Ax^* + \varphi(y, u) + L(y^* - \hat{y}^*) \quad L \in \mathbb{R}^{4 \times 3},
\]
\[
\dot{\hat{y}}^* = Cx^*.
\] (10)
The error dynamics are specified by
\[
\dot{e}^* = (A - LC)e^* \quad e^* \triangleq x^* - \hat{x}^*.
\]
As the pair \((C, A)\) is observable, the eigenvalues of the error system may be arbitrarily assigned. Note that the transformations (6) and (7) are globally invertible and the estimate \(\hat{x}\) is defined by
\[
\hat{x}_1 = \hat{x}_1^* - (K_2/N_1) \cos(N_1\hat{\theta}_4)
\]
\[
\hat{x}_2 = \hat{x}_2^* - (K_2/N_1) \sin(N_1\hat{\theta}_4)
\]
\[
\hat{x}_3 = \hat{x}_3^* - K_4\hat{x}_4^*
\]
\[
\hat{x}_4 = \hat{x}_4^*.
\] (11)
With \(A - LC\) stable, the estimate \(\hat{x}\) is guaranteed to converge to \(x\) for arbitrary initial conditions \(\delta x(0)\).

B. Derivation of the Output and State-Space Transformations for the Observer

In this subsection, the transformation (6) and (7) are derived from the general theory given in [9]. Following the notation of Krener and Respondek [9], define \(\xi_1 = \psi(x) = x_4, \xi_2 = \psi(x) = x_4, \xi_1 = \psi(x) = x_4, \xi_1 = \psi(x) = x_4\) and \(\xi_1 = \psi(x) = x_4, \xi_1 = \psi(x) = x_4, \xi_1 = \psi(x) = x_4, \xi_1 = \psi(x) = x_4\). Then, the system equations (2) become
\[
\begin{align*}
\frac{d\xi_1}{dt} &= \xi_{12} \\
\frac{d\xi_2}{dt} &= -K_3 \xi_{21} \sin(N_1\xi_{11}) \\
&\quad + K_3 \xi_{23} \cos(N_1\xi_{11}) - K_4 \xi_{12} \triangleq f_3(x)
\end{align*}
\] (12)
\[
\begin{align*}
\frac{d\xi_1}{dt} &= -K_1 \xi_{12} + K_2 \xi_{22} \sin(N_1\xi_{11}) \\
&\quad + u_1 \triangleq f_3(x) + u_1
\end{align*}
\] (13)
\[
\begin{align*}
\frac{d\xi_1}{dt} &= -K_1 \xi_{12} - K_2 \xi_{22} \cos(N_1\xi_{11}) \\
&\quad + u_2 \triangleq f_3(x) + u_2.
\end{align*}
\] (14)
In these coordinates, the system is in nonlinear observable form with observability indices \(l_2 = 2, l_2 = 1, l_1 = 1\). However, \(f_3(x)\) and \(f_3(x)\) are not in special observable form [9] as they both contain \(\xi_{12} \in \mathbb{R}^3(x)\), but \(\xi_{12} \notin \mathbb{R}^3(x)\) since the degree \(\xi_{12} = 1\) and \(l_2 = 1, l_1 = 1\).

To bring the system into special observable form, remark 4.4 and theorem 4.2 in [9] are used. As shown there, the vectors \(Y^1, Y^2, Y^3\) defined on the output coordinates, are found by solving

\[
\mathcal{L}_{Y^i}(\psi_i) = \langle d\psi_i, Y^j \rangle = \begin{cases}
1 & \text{if } i = j \\
0 & \text{otherwise}
\end{cases}
\]
(15)
where \(d\psi_{i,l_i+1} = \partial \psi_i / \partial \xi_{i,l_i+1}, d\psi_{i} = [1 \ 0 \ 0], d\psi_{i} = [0 \ 1 \ 0]\) and \(d\psi_{i} = [0 \ 0 \ 1]\). In particular, for \(i = 2, j = 1 \) and \(l_2 + 1 = 2\), it follows that
\[
f_{2,12} = \partial f_2 / \partial \xi_{12} = K_2 \sin(N_1\xi_{11}) = K_2 \sin(N_1\psi_1)
\]
and for \(i = 3, j = 1, l_3 + 1 = 2, \)
\[
f_{3,13} = \partial f_3 / \partial \xi_{13} = -K_2 \cos(N_1\xi_{11}) = -K_2 \cos(N_1\psi_3).
\]
Finally, \(f_{i,1,l_i+1} = 0\) for all other \(i, j\). The conditions (15) become
\[
\begin{bmatrix}
d\psi_1 \\
d\psi_2 \end{bmatrix} \begin{bmatrix}
Y^1 & Y^2 \\
Y^3 & Y^4 \\
\end{bmatrix} = \begin{bmatrix}
K_2 \sin(N_1\psi_1) & 1 \\
-K_2 \cos(N_1\psi_1) & 0
\end{bmatrix}.
\]
Solving this results in \(Y^1(\psi) = [1 \ K_2 \sin(N_1\psi_1) - K_2 \cos(N_1\psi_1)]^T, Y^2(\psi) = [0 \ 1]^T, Y^3(\psi) = [0 \ 0]^T\). A change of output coordinates given by
\[
\hat{\psi}_1(\psi) = \hat{\psi}_1(\psi_1, \psi_2, \psi_3)
\]
\[
\hat{\psi}_2(\psi) = \hat{\psi}_2(\psi_1, \psi_2, \psi_3)
\]
\[
\hat{\psi}_3(\psi) = \hat{\psi}_3(\psi_1, \psi_2, \psi_3)
\]
will put the system into special observable form [9] if
\[
\mathcal{L}_{Y^i}(\hat{\psi}_i) = 0 \quad \text{for } i = 2, 3.
\]
That is,
\[
\begin{align*}
\frac{d\tilde{\psi}_2}{d\tilde{\psi}_1} + K_2 \sin (N_1 \tilde{\psi}_1) \frac{d\tilde{\psi}_2}{d\tilde{\psi}_1} + K_2 \cos (N_1 \tilde{\psi}_1) \frac{d\tilde{\psi}_2}{d\tilde{\psi}_1} &= 0 \\
\frac{d\tilde{\psi}_3}{d\tilde{\psi}_1} + K_2 \sin (N_1 \tilde{\psi}_1) \frac{d\tilde{\psi}_3}{d\tilde{\psi}_1} + K_2 \cos (N_1 \tilde{\psi}_1) \frac{d\tilde{\psi}_3}{d\tilde{\psi}_1} &= 0.
\end{align*}
\]  
\tag{14}

A solution is given by
\[
\begin{align*}
\tilde{\psi}_1 &= \psi_1 \\
\tilde{\psi}_2 &= \psi_2 + (K_2/N_1) \cos (N_1 \psi_1) \\
\tilde{\psi}_3 &= \psi_3 + (K_2/N_1) \sin (N_1 \psi_1).
\end{align*}
\tag{15}

Defining a new set of coordinates on the system (2) by
\[
\begin{align*}
\eta_{11} &= \tilde{\psi}_1 - \psi_1, \\
\eta_{12} &= \tilde{\psi}_2 - \psi_2 + (K_2/N_1) \cos (N_1 \psi_1), \\
\eta_{13} &= \tilde{\psi}_3 - \psi_3 + (K_2/N_1) \sin (N_1 \psi_1)
\end{align*}
\]
then, with \(e = x - \hat{x}\), the error dynamics are \(\dot{e} = (A - lC)e\). As the pair
\[
\begin{bmatrix}
0 & 1 \\
-K_4 & 0
\end{bmatrix}
\begin{bmatrix}
l_1 \\
l_2
\end{bmatrix}
\] is observable, it is then straightforward to find \(l\) to arbitrarily assign eigenvalues of the error dynamics. In fact, a first-order observer may be constructed in the usual way [17, p. 281]. That is, let \(x = x_1 - \xi_4, l \in \mathbb{R}\), so that
\[
\begin{align*}
\dot{x}_1 &= -K_4 x_1 + K_2 x_2 \cos (N_1 x_1) \\
&\quad - K_4 (\dot{x}_3 - (K_4 + l)(x - \xi_4))
\end{align*}
\]
whose estimator is given by
\[
\dot{\hat{x}}_1 = -K_4 \hat{x}_1 + K_2 \hat{x}_2 \cos (N_1 \hat{x}_1) \\
&\quad - K_4 (\hat{x}_3 - (K_4 + l)(\hat{x} - \xi_4)).
\]
The error dynamics \(\dot{e} = x - \hat{x}\) is governed by
\[
\dot{e} = -(K_4 + l)e.
\]

Finally, the analysis for a reduced-order observer becomes simplest in the DQ representation (5). With \(l_q\) a known measurement (as \(l_1, l_2, \text{ and } \theta\) are measured), the last two equations of (5) are linear with \(\theta\) the measurement.

IV. EXPERIMENTAL RESULTS

The hardware used to implement the observer algorithm consists of the following: Motorola’s Advanced Development System (ADS) including the DSP56001 digital signal processor, a DSP data acquisition board, two amplifiers and a 50 pole PM stepper motor with a 2000 line optical encoder.

A least-squares identification algorithm was performed to determine the motor parameters [19] [20] [21]; the results were \(L = 1.31 \, mH, R = .58 \, \Omega, J = 3.32 \times 10^{-5} \, Kg \, m^{2}, K_m = .23 \, N \cdot m/A, \) and \(B = 8.0 \times 10^{-4} \, N \cdot m/s/rad\).

The observer gain matrix \(L\) was chosen to be of the form
\[
L = \begin{bmatrix}
l_{11} & 0 & 0 \\
0 & l_{22} & 0 \\
0 & 0 & l_{33}
\end{bmatrix} \in \mathbb{R}^{3 \times 3}
\]
so that \(\text{det}(A - LC) = (s + l_1)x_1 + l_2x_2(s^2 + l_3x_1 + l_3x_2).
\]

In all plots below, the horizontal scale is time in seconds. The actual measurement is indicated by a solid line while the estimate is indicated by a dashed line.

In the first set of experiments, linear amplifiers \((\approx 1200 \, Hz\, bandwidth)\) were used along with a sampling rate of 10 kHz. The observer gains were chosen as \(l_{11} = 5 \times 10^{-3}, l_{22} = 5 \times 10^{-3}, l_{13} = 7 \times 10^{-5}, l_{33} = 5.3 \times 10^{-5}\). As shown in Fig. 1, the motor was accelerated to a speed of 720 rpm and then taken back down to rest. Note the excellent tracking of the estimate \(\hat{\omega}\) to the actual speed \(\omega\) in Fig. 1. The actual speed was found off-line by computing \((\theta_dot(k + 1)/T) - \theta_k/T)\) \(T\) and passing this through a second-order noncausal filter (fitfill in MATLAB) to remove the noise.

The phase current \(i_{2}\) and its estimate \(\hat{i}_{2}\) are shown below in Fig. 2. Again, as for the speed, the estimate \(\hat{i}_{2}\) tracks \(i_{2}\) quite well. Finally, Fig. 3 below shows the estimation error \(i_{2} - \hat{i}_{2}\).

The phase current \(i_{1}\) and its estimate \(\hat{i}_{1}\) had similar responses as phase \(a\) and thus are not shown.

In the second set of experiments, 20 kHz pulse-width modulated (PWM) amplifiers were used along with a sample rate of 50
kHz. The observer gains were chosen as $l_{11} = 2.5 \times 10^4$, $l_{22} = 2.5 \times 10^4$, $l_{33} = 7 \times 10^5$, $l_{43} = 5.3 \times 10^3$. The speed and its estimate are shown in Fig. 4. Again, note the excellent speed estimation. The phase current $i_u$ and its estimate $\hat{i}_u$ are shown in Fig. 5. Fig. 6 is zoomed version of Fig. 5 in order to show the
estimation in more detail. The phase current $i_p$ and its estimate $\hat{i}_p$ had similar responses. The actual speed $\omega$ was computed the same way as in the previous experiment.

V. CONCLUSIONS

The PM stepper motor was shown to be an interesting application of a multi-output observer with linear error dynamics. A nontrivial output transformation was found using the Krener-Respondek theory in order to construct a full-order observer with linear error dynamics. The practicality of the observer algorithm was demonstrated with an actual implementation.

ACKNOWLEDGMENTS

The authors would like to thank Martin Corless for helpful discussions. They are also grateful to Motorola for donating the DSP56001 Advanced Development System used in this research.

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Dynamic Feedback Linearization of the Induction Motor

John Chiasson

Abstract—This work develops the use of dynamic state feedback to achieve feedback linearization of the induction motor. It is shown that the sixth-order nonlinear dynamical model of the induction motor consisting of rotor speed, two stator currents, two rotor fluxes and an integrator in the feedback controller can be made equivalent to two third-order decoupled linear systems by nonlinear state feedback through the stator input voltages. It is further shown that a nonsingular dynamic feedback linearizing transformation exists as long as the (electromagnetic) torque put out by the motor is nonzero.

I. INTRODUCTION

The induction motor is the motor of choice in many industrial applications due to its reliability, ruggedness and relatively low cost. Its mechanical reliability is due to the fact that there is no mechanical commutation (i.e., there are no brushes nor commutator to wear out as in a DC motor). Furthermore, it can also be used in volatile environments since no sparks are produced as is the case in the commutator of a DC motor. The induction motor is named (appropriately) after the fact that the currents in the rotor are electromagnetically induced by a rotating magnetic field set up by the stator. It is these induced currents that interact with the aforementioned rotating stator magnetic field to produce torque and, because of this, there is no need for brushes and a commutator as in the DC motor.

The induction motor presents an extremely challenging control problem. This is due primarily to three issues: i) the dynamic model of the system is nonlinear, ii) two of the state variables (rotor fluxes) are not usually measurable, and iii) due to ohmic heating, the rotor resistance varies considerably with a corresponding significant effect on the system dynamics. The first issue just enumerated is considered in this work.

The present state of the art in induction motor control is the so-called field-oriented vector control technique introduced by Blaschke [1]. This method consists in rewriting the dynamic equations of the induction motor in a reference frame that rotates with the magnetizing “current vector” (i.e., a time-varying complex phasor representation of the magnetizing current). In this new coordinate system, insight is given on how to regulate the magnetizing current in order to keep the stator

Manuscript received September 20, 1991; revised August 28, 1992.
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IEEE Log Number 9208720.