Estimating the State of Charge of a Battery

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Abstract—This work considers the state of charge (SOC) estimation problem for electrochemical batteries. Using an electric circuit model of the battery given in the literature, it is shown how the open circuit voltage (which is directly related to the SOC) can be estimated based on the terminal voltage and current measurements provided there is sufficient variation in the battery current.

Keywords—Battery, State of Charge, Observability Gramian, Linear Time Varying Systems

I. INTRODUCTION

In electric vehicles, a key parameter is the state of charge of the battery as it is a measure of the amount of electrical energy stored in it. It is analogous to a fuel gauge on a conventional internal combustion (IC) car. To define the state of charge, consider a completely discharged battery. With \( I_b(t) \) the charging current, the charge delivered to the battery is \( \int_{I_0}^{I_b(t)} I_b(\tau) d\tau \). With \( Q_0 = \int_{I_0}^{\infty} I_b(\tau) d\tau \) the total charge the battery can hold, the state of charge (SOC) of the battery is simply

\[
SOC(t) = \frac{\int_{I_0}^{I_b(t)} I_b(\tau) d\tau}{Q_0} \times 100. \tag{1}
\]

Typically, it is desired that the state of charge of the battery be kept within appropriate limits, for example \( 20\% \leq SOC(\%) \leq 95\% \). As a consequence, it is essential to be able to estimate the state of charge of the battery to maintain the state of charge within safe limits. Estimating the battery state of charge (SOC) is not an easy task because the SOC depends on many factors such as temperature, battery capacitance and internal resistance. One possible way to estimate the SOC is by direct application of (1), but this is subject to biases as it is a pure integration. Another approach is to compute the open circuit voltage of the battery (the voltage when the battery current is zero). It has been shown that there is a linear relationship between the state of charge of the battery and its open circuit voltage given by [1]

\[
\begin{align*}
V_{oc}(t) &= a_1 S(t) + a_0 \\
S(t) &= \frac{V_{oc}(t) - a_0}{a_1} \tag{2}
\end{align*}
\]

where \( S(t) \) is the state of charge (%) of the battery, \( a_0 \) is the battery terminal voltage when \( S(t) = 0\% \), and \( a_1 \) is obtained knowing the value of \( a_0 \) and \( V_{oc} \) at \( S(t) = 100\% \). By (2), the estimation of the state of charge is equivalent to the estimation of its open circuit voltage. However, in order to measure the open circuit battery voltage \( V_{oc} \), the battery must be disconnected from the load which is not possible during vehicle operation.

II. BATTERY TERMS

There are several parameters associated with battery modeling, and the parameters which are relevant to model used here are briefly described [2].

Self-discharge resistance: It is the resistance that is associated with the electrolysis of water at high voltage levels and slow leakage across the battery terminal at low voltage. This resistance is more temperature sensitive and is inversely proportional to the temperature [2].

Charge and discharge resistance \( (R_c/R_d) \): These resistances are associated with the electrolyte resistance, plate resistance, and fluid resistance and represent the fact that these values differ depending on whether the battery is charging or discharging.

Overcharge and over discharge resistance: These resistances are attributed largely to the electrolyte diffusion during over charging and over discharging.

Polarization capacitance \( (C) \): This is the capacitance used to model the chemical diffusion of the electrolyte within the battery (rather than a purely electrical capacitance). It depends on SOC, temperature and also the device design.

Continuous discharging: In this case, the battery continuously delivers energy to the load which leads to a continuous drop in the battery capacity.

Intermittent discharging: In this case, the battery delivers energy to the load at regular or irregular intervals of time. This is typical in HEVs where the energy is drawn by the motor for some period followed by the voltage recovery period.

Rate of charge and discharge: To extend service life of the battery, the rate of charge or discharge should not be too high. Also the frequency of charging and discharging
cycles affect the battery life significantly. The frequency of switching between charging and discharging is especially high in electric and hybrid electric vehicles which reduces the life of the battery.

III. Battery models

A commonly used battery model is shown in Figure 1. It consists of an ideal battery with open-circuit voltage \( V_{oc} \), a constant equivalent internal resistance \( R_{int} \) and the battery terminal voltage represented by \( V_b \). The terminal voltage \( V_b \) can be obtained from the open circuit measurement, and \( R_{int} \) can be measured by connecting a load and measuring both the terminal voltage and current, at fully charged condition. However, it has been found that the internal resistance is different under discharge and charge conditions. Also, this model does not capture the internal dynamics of the battery, in particular the effect of the diffusion of the electrolytic chemicals between the battery plates.

![Fig. 1. Simple battery model (see [1]).](image)

To account for the different resistance values under charge and discharge conditions, the circuit can be modified as shown in 2.

![Fig. 2. Battery model accounting for the different charging and discharging resistance values (see [1]).](image)

This model has two kinds of internal resistances, \( R_c \) and \( R_d \), which are associated with the charging and discharging process of the battery, respectively. These two parameters \((R_c \text{ and } R_d)\) model all forms of energy loss which includes electrical and non-electrical losses. The diodes, shown in Figure 2, implies that during charging or discharging only one of the resistances \( R_c \) or \( R_d \) (which is in series with the forward biased diode) will be used because when one diode is forward biased the other will be reverse biased. These diodes are present only for modeling purposes only and have no physical significance in the battery.

In order to model the diffusion of the electrolytic through the battery and its resultant effect of causing transient currents in the battery, a capacitor is added to the model as shown in Figure 3 from [1][3]. This is the model adopted here to develop a state of charge estimation scheme.

![Fig. 3. Battery model with polarization capacitance as well as \( R_c, R_d \) (see [1]).](image)

The dynamic equations of the circuit model for discharging and charging are given by,

\[
\dot{V_p} = -\frac{1}{R_cC}V_p + \frac{1}{R_cC}V_{oc} - \frac{1}{C}I_b, \quad V_p \leq V_{oc} \quad (3)
\]

\[
\dot{V_p} = -\frac{1}{R_dC}V_p + \frac{1}{R_dC}V_{oc} - \frac{1}{C}I_b, \quad V_p > V_{oc} \quad (4)
\]

where

\[
I_b = \frac{V_p - V_{oc}}{R_b}
\]

The current \( I_b \) is considered to have a positive sign when the battery is discharging. As explained in [1], the capacitance \( C \) represents a “polarization capacitance” and models the chemical diffusion within the battery. Its value depends on \( SOC \), temperature and also the device design. This particular circuit model is chosen to model internal workings of the battery for the following reasons: It accounts for the electrical and non-electrical energy losses (during charging and discharging of the battery) through the choice of the circuit parameters \( R_d \) and \( R_c \) and models the transient behavior of the internal battery current (especially important in the HEV and EV operation) by including the polarization capacitance \( C \).

None of the parameters \( R_c, R_d, C \) are known a priori and \( V_p \) is not measurable. The problem then is to estimate \( V_{oc} \) (the SOC is then found using (2)) with only measurements of the terminal voltage and current.

Of course, the model (3) (4) is not the only model that has been proposed for batteries. Circuit models in the same spirit as the considered here have been proposed in [4][5][6]. A model based on the kinetic reactions and diffusion process inside the battery has recently been proposed in [7].
IV. Mathematical Model

In this section, the dynamic equations of the model in Figure 3 as given by (3) and (4), are expanded to account for the fact that circuit parameters are unknown. The open circuit voltage, internal capacitor voltage and the terminal voltage are represented by $V_{oc}$, $V_p$ and $V_b$ respectively. The charging, discharging and the internal resistance of the battery are represented by $R_c$, $R_d$, and $R_b$, respectively; and the polarization capacitance of the battery is represented by $C$. The current $I_b$ is taken as positive if discharging and negative otherwise. Consider the case when the battery is discharging (the charging case is similar). The loop equations pertaining to the circuit model are represented by equation (5).

$$
V_p = \frac{1}{C} \left( (V_{oc} - V_p) / R_d - I_b \right)
$$

(5)

Here the state space model of [1] is used. This model is found by defining the state variables

$$
x_1 = V_p; x_2 = \frac{1}{R_dC}; x_3 = \frac{V_{oc}}{R_dC}; x_4 = \frac{1}{C}; x_5 = R_b
$$

(6)

so that the nonlinear time-varying state space model is then

$$
\dot{x}_1 = -x_1 x_2 + x_3 - I_b(t) x_4

\dot{x}_2 = 0

\dot{x}_3 = 0

\dot{x}_4 = 0

\dot{x}_5 = 0

V_b = x_1 - I_b(t) x_5
$$

(7)

where $x(0) = (x_{10}, x_{20}, x_{30}, x_{40}, x_{50})^T$ is unknown. The fundamental question is whether or not the open circuit voltage $V_{oc} = x_{30}/x_{20}$ can be determined from the terminal voltage $V_b$ and current $I_b$. That is, can one estimate the initial value of the state variables $x_{20}, x_{30}$ from knowledge of the output $V_b$, input $I_b$ and the system model (7)? In [1], an extended Kalman filter approach was used. In [1], a noise model was added to (7), the model was then linearized so that one works with a linear time-varying system for which the standard Kalman filter can be applied. (One of the beautiful things about the Kalman filter is that its formulation is no more difficult for the linear time-varying case than for the linear time-invariant case.) As pointed out by Farrell [8], the system cannot be observable if the battery current is constant. Consequently, the approach here is to consider a deterministic linear time-varying model to estimate $V_{oc}$. Specifically, the system (7) is viewed as a linear time-varying system with the unknown parameter $x_{20}$. To proceed, let $z_1 = x_1, z_2 = x_3, z_3 = x_4, z_4 = x_5$ and consider the system

$$
\frac{dz}{dt} = \begin{bmatrix}
-x_{20} & 1 & -I_b(t) & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
z_1(t) \\
z_2(t) \\
z_3(t) \\
z_4(t)
\end{bmatrix}
$$

$$
z(t_0) = \begin{bmatrix}
x_{10} \\
x_{30} \\
x_{10} \\
x_{50}
\end{bmatrix}
$$

(8)

$$
y(t) = \begin{bmatrix}
1 & 0 & 0 & -I_b(t)
\end{bmatrix} \begin{bmatrix}
z_1(t) \\
z_2(t) \\
z_3(t) \\
z_4(t)
\end{bmatrix}
$$

(9)

It is easy to check that if the current $I_b$ is constant in the system model (8), the resulting linear time-invariant system is not observable. In fact, as will be shown below, with the battery current of the form $I_b(t) = a + b(t - t_0) + c(t - t_0)^2$, the system (8) is observable if and only if $c \neq 0$. Consequently, the time-varying behavior of the current $I_b(t)$ is essential to the system being observable.

V. Observability Gramian

In compact form, the system (8) may be rewritten as

$$
\frac{dz}{dt} = A(t, x_{20}) z(t), \quad z(t_0) = (x_{10}, x_{30}, x_{40}, x_{50})
$$

$$
y(t) = C(t) z(t)
$$

with the obvious definitions for $A(t, x_{20})$ and $C(t)$. If this system satisfies certain observability criteria, then one can estimate $z(t_0) = (x_{10}, x_{30}, x_{40}, x_{50})$ from knowledge of the output $y(t)$ and the input $I_b(t)$. Then the open circuit voltage is found from $V_{oc} = x_{30}/x_{20}$ (The fact that $x_{20}$ is unknown will be dealt with later).

By definition [9], the unforced linear state equation given by (9) is observable on $[t_0, t_f]$ if any initial state $z(t_0)$ is uniquely determined by its corresponding response $y(t)$, for $t \in [t_0, t_f]$. The parameter of concern here is the ratio $x_{30}/x_{20} = V_{oc}$. The state transition matrix of the system defined by equation (9) is defined as the solution to

$$
\frac{d\Phi(t, t_0, x_{20})}{dt} = A(t, x_{20}) \Phi(t, t_0, x_{20})
$$

$$
\Phi(t_0, t_0, x_{20}) = I_{n \times n}
$$

(10)

Then

$$
z(t, t_0, x_{20}; z_0) = \Phi(t, t_0, x_{20}) z(t_0)
$$

$$
y(t) = C(t) \Phi(t, t_0, x_{20}) z(t_0)
$$

(9)

In order to calculate $z(t_0)$ this system must be observable. The linear state equation given by equation (9) is observable on $[t_0, t_f]$ if and only if the $n \times n$ Gramian matrix $M$ defined by

$$
M(t_0, t_f, x_{20}) \triangleq \int_{t_0}^{t_f} \Phi^T(t, t_0, x_{20}) C^T(t) C(t) \Phi(t, t_0, x_{20}) dt
$$

(11)
is invertible. If \( M(t_0, t_f, x_{20}) \) is invertible, then the initial state \( z(t_0) \) is found by multiplying \( y(t) \) by \( \Phi^T C^T \) and integrating to get (see [9])

\[
\int_{t_0}^{t_f} \Phi^T(t, t_0, x_{20})C^T(t)y(t)\,dt
\]

\[
= \int_{t_0}^{t_f} \Phi^T(t, t_0, x_{20})C^T(t)C(t)\Phi(t, t_0, x_{20})z(t_0)\,dt
\]

\[
= M(t_0, t_f, x_{20})z(t_0)
\]

Then

\[
z(t_0) = M^{-1}(t_0, t_f, x_{20}) \int_{t_0}^{t_f} \Phi^T(t, t_0, x_{20})C^T(t)y(t)\,dt
\]

(12)

The ability to compute \( z(t_0) \) relies on the Gramian being nonsingular. To find conditions under which the Gramian is nonsingular, the fundamental solution is first computed and then used to compute the Gramian. Solving (10) for \( \Phi(t, t_0, x_{20}) \) reduces to

\[
\frac{d\Phi_{11}}{dt} = -\Phi_{11}x_{20}
\]

(13)

\[
\frac{d\Phi_{12}}{dt} = -\Phi_{12}x_{20} + 1
\]

(14)

\[
\frac{d\Phi_{13}}{dt} = -\Phi_{13}x_{20} - I_b(t)
\]

(15)

\[
\Phi_{14} \equiv 0
\]

(16)

and

\[
\Phi_{ij} = \delta_{ij}, i = 2, 3, 4 \text{ and } j = 1, \ldots, 4.
\]

(17)

Then

\[
\Phi(t, t_0, x_{20}) = \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where \( \Phi_{11} = e^{-x_{20}(t-t_0)} \), \( \Phi_{12} = \frac{1}{x_{20}}(1 - e^{-x_{20}(t-t_0)}) \) and \( \Phi_{13} = -\int_{t_0}^{t} e^{-x_{20}(t-\tau)}I_b(\tau)\,d\tau. \) As stated previously, the key to observability of the system is the time variation of the current \( I_b(t) \). Consider that over any short time interval \([t_0, t_f]\), the battery current waveform can be fit to a quadratic equation of the form

\[
I_b(t) = a + b(t - t_0) + c(t - t_0)^2
\]

(18)

for some parameter values \( a, b \) and \( c \). Then

\[
\Phi_{13}(t, t_0, x_{20}) = -\int_{t_0}^{t} e^{-x_{20}(t-\tau)}I_b(\tau)\,d\tau
\]

\[
= \left(\frac{2c - bx_{20} + ax_{20}^2}{x_{20}^3}\right) e^{-x_{20}(t-t_0)}
\]

\[
-\frac{1}{x_{20}^4} \left(2c - bx_{20} + ax_{20}^2 - 2cx_{20}(t - t_0) - b^2x_{20}^2(t - t_0) + c^2x_{20}(t - t_0)^2\right)
\]

Using this expression for \( \Phi_{13}(t, t_0, x_{20}) \) along with (17)(13)(14)(15)(16), the observability Gramian \( M \) is given by

\[
M(t_0, t_f, x_{20}) = \int_{t_0}^{t_f} \begin{bmatrix}
\Phi_{11}^2 & \Phi_{11}\Phi_{12} & \Phi_{11}\Phi_{13} & -\Phi_{11}I_b \\
\Phi_{12}\Phi_{11} & \Phi_{12}^2 & \Phi_{12}\Phi_{13} & -\Phi_{12}I_b \\
\Phi_{13}\Phi_{11} & \Phi_{13}\Phi_{12} & \Phi_{13}^2 & -\Phi_{13}I_b \\
-I_b\Phi_{11} & -I_b\Phi_{12} & -I_b\Phi_{13} & I_b^2
\end{bmatrix} \,dt
\]

(19)

This computation was carried out analytically using MATHEMATICA [10]. The important fact here is that the Gramian has full rank if and only if \( c \neq 0 \) in (18). In the case \( c = 0 \), the Gramian has full rank for any \( t_f > t_0 \).

The open circuit voltage \( V_{oc} = x_{30}/x_{20} \) is required to estimate the state of charge, but \( x_{20} \) is unknown. However, again using MATHEMATICA, the computation

\[
\frac{x_{30}}{x_{20}} = \frac{1}{x_{20}} \left(\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} M^{-1}(t_0, t_f, x_{20}) \times \int_{t_0}^{t_f} \Phi^T(t, t_0, x_{20})C^T(t)y(t)\,dt\right)
\]

(20)

was performed. It was found that the right-hand side of (20) had terms containing \( e^{-x_{20}(t_f-t_0)}, e^{-2x_{20}(t_f-t_0)} \). If these terms are set to zero, it turns out the ratio \( x_{30}/x_{20} \) is independent of \( x_{20} \) ! In other words, \( V_{oc} = x_{30}/x_{20} \) is asymptotically independent of \( x_{20} \) as \( e^{-x_{20}(t_f-t_0)} \to 0, e^{-2x_{20}(t_f-t_0)} \to 0 \) as \( t_f - t_0 \to \infty \). Mathematically,

\[
V_{oc} = \lim_{t_f - t_0 \to \infty} \frac{x_{30}}{x_{20}}
\]

\[
= \lim_{t_f - t_0 \to \infty} \left(\frac{1}{x_{20}} \left[ 0 \ 1 \ 0 \ 0 \right] M^{-1}(t_0, t_f, x_{20}) \times \int_{t_0}^{t_f} \Phi^T(t, t_0, x_{20})C^T(t)y(t)\,dt\right)
\]

(21)

does not depend on \( x_{20} \). The approach here is to use a pretty much arbitrary value \( \bar{x}_{20} \) for the unknown value \( x_{20} \) in (21) to compute \( V_{oc} \). This procedure is outlined in the next section.

Remark 1: This result guarantees the open circuit voltage \( V_{oc} \) can be (asymptotically) estimated without knowledge of any of the circuit parameters. This could explain why the extended Kalman filter estimation algorithm in [1] seems to estimate \( V_{oc} \) while not accurately estimating \( R_d \) (see Figure 6a in [1] and recall that \( x_{20} = 1/(R_dC) \)). Though one is not typically interested in the value of \( R_d \), in contrast to [1], the approach here guarantees the (asymptotic) estimate of \( V_{oc} \) without knowing \( R_d \) or any other circuit parameter.

VI. ESTIMATION ALGORITHM

The estimation algorithm for the open circuit voltage \( V_{oc} \) is given by

1. Sample the current \( I_b \) over an interval \([t_0, t_f]\)
2. Use a least-squares algorithm to compute \( a, b, c \) to fit the samples of the current to the expression (18).
3. Test that $c \neq 0$ to ensure the Gramian is full rank. If it is zero, then do not estimate $V_{oc}$ over that time interval.
4. Using a “guess-estimate” value $\hat{x}_{20}$ of the actual value $x_{20} = \frac{1}{2C}R_d$, compute $V_{oc} = \lim_{t \to \infty} \frac{1}{x_{20}}$ using the right-hand side of (21).

**Remark 2:** The ratio $\frac{x_{30}}{x_{20}}$ is independent of the value of $x_{20}$ only after $e^{-x_{20}(t_f-t_0)} \to 0$ with the actual (but unknown) value of $x_{20}$. Consequently, knowledge of the order of magnitude of $x_{20}$ is needed so that one can be assured that $e^{-x_{20}(t_f-t_0)} \to 0$ in the time interval $[t_0, t_f]$.

**VII. Experimental Results**

The experiments reported here were carried out using the ABC-150 [11] bidirectional battery charger/discharger. This device has been designed specifically for testing electric and hybrid-electric vehicle batteries. It is controlled through a personal computer (PC) using a special operating system known as ABC-150 ROS (Remote Operating System). ROS is a PC based graphical user interface (GUI) application and runs on the Windows NT operating system. The ABC-150 is a test system for a wide range of DC loads and has the ability to follow a user defined current, voltage or power profile within an allowable range.

The tests were conducted for different load conditions using the ABC-150 and standard 12 Volt 16 Ampere-Hour lead acid battery made by HAWKER GENESIS [12]. In these experiments, the unknown parameter $x_{20} = \frac{1}{2C}R_d$ was set equal to 1/3 (The values $R_d = 7.5$ m$\Omega$ from [12] and $C = 40$ F from [1] were used). Of course, the actual open circuit voltage is not known. However, when the battery current $I_b$ is zero, the terminal voltage $V_b$ and open circuit voltage $V_{oc}$ should be equal. Further, if the battery is discharging ($I_b > 0$) then $V_{oc} > V_b$ while if the battery is charging, $V_b > V_{oc}$.

Figure 4 is a current waveform chosen so that it goes to zero for short intervals of time. This allows one to check if $V_{oc} = V_b$ during these intervals as well as check that $V_{oc} > V_b$ when $I_b > 0$. Figure 5 shows that the estimated $V_{oc}$ is close to $V_b$ during the time intervals that $I_b$ is zero and $V_{oc} > V_b$ when $I_b > 0$.

Another test was performed using the current profile given in Figure 6. The corresponding battery terminal voltage and estimated open circuit voltage (based on the above algorithm) are given in Figure 7.

**VIII. Conclusions and Summary**

The objective of this work was to estimate the state of charge of lead-acid batteries. A modified Thevenin equivalent circuit model given in [1] was used to represent the lead-acid battery. Here the approach was to treat the nonlinear time varying model as a linear time varying model with an unknown constant parameter $x_{20}$. Conditions were found on the battery current that ensure the observability Gramian of the system is full rank so that the initial state of the system can be found using the inverse of the system Gramian. The open circuit voltage is given by the ratio $V_{oc} = x_{30}/x_{20}$ where it was shown that this ratio was independent of $x_{20}$ as long as $e^{-x_{20}(t-t_0)} \approx 0$, that is, after a short time interval.

Future work would include trying the above methodology on the models proposed in [4][6].

**IX. Acknowledgements**

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Battery Discharging Current (A)

Battery Terminal Current $I_b(t)$ (discharging)

Test using three 12 Volt batteries connected in series

References


