High-Speed Parameter Estimation of Stepper Motors

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Abstract—A batch least-squares algorithm is applied to identify the parameters of a permanent magnet stepper motor. Short data segments of less than a tenth of a second are considered and the procedure does not require a special configuration of the motor. Error indexes are studied as measures of uncertainty in the estimation procedure. Simulations are performed to test the identification method and the usefulness of the error indexes. Experimental results, obtained on an industrial setup, demonstrate the validity of the approach in an actual application. The results can be used to develop models of stepper motors for precise control and to automatically tune computer-controlled drives.

I. INTRODUCTION

RECENTLY, there has been an increase in the use of ac motors in positioning applications. This increase is attributed to developments in power electronics and in computing technology. The faster computing capabilities have made it possible to perform more complex calculations in a shorter period of time. These advances have opened up new opportunities for advanced control methods such as nonlinear and optimal control. In addition, new opportunities exist for self-tuning and adaptive methods to determine known or slowly varying parameters. These methods are useful to automatically adjust the control system to maximize performance with minimal or no operator intervention.

The objective of the research presented in this paper is to study on-line estimation methods for stepper motors that are fast and implementable in contemporary high-performance hardware. Traditionally, dc motors have been chosen for industrial applications over stepper motors due to their linear input/output relationship which, in turn, allows for the use of standard control strategies. Over the past few years, the use of stepper motors has increased. The reasons for this include: better reliability due to the elimination of mechanical brushes, better heat dissipation as the windings are located on the stator and not on the rotor, higher torque-to-inertia ratio due to a lighter rotor, and lower prices.

Stepper motors were originally designed to be used in open-loop. Their inherent stepping ability allows for accurate positioning without feedback. An example of optimal open-loop control for a variable-reluctance stepper motor was demonstrated in [6] using a conjugate gradient method. Closed-loop control of stepper motors has been used increasingly in the last decade to achieve faster response times and higher resolution capabilities [1], [7]. Adaptive control was successfully demonstrated in [5] to achieve higher precision by cancelling torque-ripple effects. The stepper motor can also be operated at higher speeds, by taking nonlinear effects into consideration [3].

The estimation method used in this paper is a batch least-squares algorithm. Its advantages are that it can be performed quickly, with short data segments, and does not require special tests or configuration of the machine. Error indexes are used for the evaluation of the procedure. These indexes provide a measure of the uncertainty in the parameter estimates. The least-squares algorithm used in this paper can be made recursive for use in adaptive control if parameters vary slowly. It is also the basis of procedures developed for the real-time identification of synchronous ac motors [11], and induction motors [13], [14].

The identification procedure is applied to an industrial setup. A good correspondence is obtained between the data and the model. The results are comparable to those obtained in simulations and in conventional tests. The method presented in this paper can be incorporated into a high-speed on-line self-tuning adaptive control scheme for use in industrial applications. The results have also been used to determine the stepper motor parameters for the implementation of a nonlinear state-space control algorithm [3].

II. DYNAMIC MODEL

A. PM Stepper Motor

We consider a two-phase permanent magnet (PM) stepper motor. A simplified schematic of a motor with one pole-pair is shown in Fig. 1. Commanded voltages \((v_a, v_b)\) control the two phase currents \((i_a, i_b)\). The magnetic field, due to the stator currents, interacts with the permanent magnet on the rotor to create a torque, so that the rotor will tend to align itself with the magnetic field produced by the currents. Applying a sequence
of voltages to each phase in succession will cause the rotor to step. The size of each step is 90 deg in the case of Fig. 1. In general, it is determined by \( N_r \), the number of rotor teeth (only teeth of the same polarity are counted so that \( N_r = 1 \) in Fig. 1). The formula for the step size \( \theta_s \) of the two-phase PM stepper motor is given by

\[
\theta_s = \frac{90}{N_r} \text{ deg.}
\]

(1)

An extensive discussion of the modeling and operation of stepper motors can be found in [1], [7].

A simple state-space model of the two-phase PM stepper motor is given by (see, e.g., [7])

\[
\begin{align*}
L \frac{di_d(t)}{dt} &= v_d(t) - R_i i_d(t) + K_m \omega(t) \sin(N_r \theta(t)) \\
L \frac{di_q(t)}{dt} &= v_q(t) - R_q i_q(t) - K_m \omega(t) \cos(N_r \theta(t)) \\
J \frac{d\omega(t)}{dt} &= -K_m i_d(t) \sin(N_r \theta(t)) + K_m i_q(t) \cos(N_r \theta(t)) - B\omega(t) - C \text{sgn} (\omega(t)) \\
\frac{d\theta(t)}{dt} &= \omega(t).
\end{align*}
\]

(2)

The model includes four state variables, two inputs, and six parameters, which are listed in Table I. Several assumptions have been made in order to obtain this simplified model, which are standard for control system design. First, the magnetic material is assumed to be linear, i.e., the effect of saturation at high currents is neglected. This assumption is reasonable because we are interested in this research in pushing the performance of the stepper motor drive toward high-speed regions, where currents are limited because of the back-EMF. The model also neglects the variation of inductance with position, i.e., assumes negligible air gap variations. Such variations are the source of the so-called reluctance torque, which is the basis of the functioning of switched reluctance stepper motors, but is negligible for permanent magnet stepper motors. Finally, the model neglects the so-called detent torque, which originates from the tendency of the permanent magnet to align itself along the directions of minimum reluctance. This is a significant effect at low speeds, but can easily be modeled and incorporated in the identification procedure described in this paper. For high-speed operation, it was found that the contribution could be neglected.

Even with these assumptions, the state-space model is nonlinear. In particular, each term associated with the motor constant \( K_m \) is the product of a state variable with \( \cos(N_r \theta) \) or \( \sin(N_r \theta) \). However, we will assume that the state variables are measurable so that the model is linear with respect to the parameters. This property allows the use of identification techniques, such as the least-squares algorithm described in Section III-A, to estimate the parameter values.

### B. DQ Transformation

Use of the model (2) to develop a least-squares parameter identifier was initiated in [4], [10]. It was found that, at high speeds, a large volume of data had to be processed by the algorithm because of the high frequencies of the signals \( i_a \) and \( i_b \), which typically vary at \( N_R \omega \) 50 times the frequency of rotation. This problem was alleviated by the use of the direct-quadrature (DQ) transformation. This transformation changes the frame of reference from the fixed phase axes to axes moving with the rotor. Equation (3) gives the DQ transformation performed on the phase currents. A similar transformation is performed on the phase voltages:

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} = \begin{bmatrix}
\cos(N_r \theta) & \sin(N_r \theta) \\
-\sin(N_r \theta) & \cos(N_r \theta)
\end{bmatrix} \begin{bmatrix}
i_a \\
i_b
\end{bmatrix}
\]

(3)

The resulting DQ coordinate state-space representation is:

\[
\begin{align*}
L \frac{di_d(t)}{dt} &= v_d(t) - R_i i_d(t) + L N_r \omega(t) i_q(t) \\
L \frac{di_q(t)}{dt} &= v_q(t) - R_i i_q(t) - L N_r \omega(t) i_d(t) - K_m \omega(t) \\
J \frac{d\omega(t)}{dt} &= K_m i_d(t) - B\omega(t) - C \text{sgn} (\omega(t)) \\
\frac{d\theta(t)}{dt} &= \omega(t).
\end{align*}
\]

(4)

This transformation has several advantages. First, the cos and sin functions have been eliminated. Second, at constant speed, \( i_d \) and \( i_q \) are constant, whereas \( i_a \) and \( i_b \) vary at \( N_R \omega \) rad/s. Also note that, for small \( \omega \), the nonlinear terms in the equations for the currents can be ignored and the model (except for the Coulomb friction term) is linear. A linear controller can then be used to control \( \omega \) using \( i_q \). For large \( \omega \), the nonlinear terms are significant but can be cancelled using the voltages \( v_d, v_q \). This is the idea of feedback linearization developed in [3], [17]. In summary, the transformation is useful both from a signal processing point of view, and from a control design point of view.

### III. IDENTIFICATION METHOD

#### A. Batch Least-Squares Identification

Using the DQ state-space model given in (4), an output equation is formed which is linear in the parameters (such an equation can also be obtained with the original variables,
but we choose the DQ model because of reasons mentioned earlier:

\[ y[n] = W^T[n]K_N. \]  

(5)

The so-called output vector \( y \), regressor matrix \( W \), and nominal parameter vector \( K_N \) are defined in (6)–(8) (see the bottom of this page). We let \( K \) be an estimate of the nominal (true) parameter vector \( K_N \). The error equation (9) is then formed by subtracting the actual output from the estimated output


(9)

If \( K = K_N \), this error would be zero in theory. In practice, however, (5) is not exactly satisfied due to modeling errors, quantization, and noise. In other words, the error \( e[n] \) is never zero. By definition, the residual error \( R_e \) is equal to the sum of the norm squared error over an interval \([N_0, N_1]\)

\[ R_e(K) = \sum_{n=N_0}^{N_1} ||e[n]||^2 \]  

(10)

\[ = \sum(W^T[n]K - y[n])^T(W^T[n]K - y[n]). \]  

(11)

The least-squares estimate \( (K^*) \) is the one that minimizes the residual error. It is found by setting the derivative of \( R_e \) with respect to \( K \) equal to zero and solving for \( K \), leading to

\[ K^* = \left( \sum W[n]W^T[n] \right)^{-1} \left( \sum W[n]y[n] \right). \]  

(12)

This algorithm is the core of most identification and adaptive control schemes (cf. [9], [12]).

A modification can be performed on the output equation to eliminate known parameters from being estimated. The known parameters \( (K_{kn}) \) and their corresponding columns in the regressor matrix \( (W_{kn}^T) \) are combined with the output vector to form a new output \( \tilde{y} \) which is linear in the remaining unknown parameters \( \tilde{K} \), so that

\[ y = [W^T \quad W_{kn}^T] \begin{bmatrix} \tilde{K} \\ K_{kn} \end{bmatrix} \]  

(13)

\[ \tilde{y} = W^T \tilde{K} \]  

(14)

where

\[ \tilde{y} = y - W_{kn}^T K_{kn} \]  

(15)

Also, a two-stage identification can be performed by breaking the output (6) into electrical and mechanical variables. The batch least-squares algorithm is first applied to the electrical variables to identify the parameters \( R, L, \) and \( K_m, \) using

\[ \begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} i_d + \frac{d}{dt} - N_r \omega i_q \\ i_q - \frac{d}{dt} + N_r \omega i_d \end{bmatrix} \begin{bmatrix} R \\ L \end{bmatrix} \]  

(16)

Then, the estimate of \( K_m \) is used in the mechanical output equation (17) to identify the remaining parameters, with

\[ [K_m i_q] = \begin{bmatrix} \frac{d}{dt} & \omega & \text{sgn}(\omega) \end{bmatrix} \begin{bmatrix} J \\ B \\ C \end{bmatrix}. \]  

(17)

One reason for performing the two-stage identification is to reduce computations. Another reason is to avoid the mixing of variables with different dimensions in the least-squares criterion. The first output equation (16) consists of two electrical equations in volts while the second (17) is a mechanical equation in radians per seconds squared. Therefore, the least-squares solution may depend on the relative scaling of the variables. The time scales for both sets of equations are also different. The electrical variables typically vary faster than the mechanical variables. Therefore, the identification could be performed with a lower sampling rate on (17).

B. Error Analysis

In any identification experiment, it is important to quantify the fitness of the data to the model and the accuracy of the parameters. Statistical measures of errors are available in the literature [12] based on the assumption that the output equation (9) is perturbed by additive noise. Typically, it is assumed that this noise is a white noise sequence, uncorrelated with the signals in the regressor matrix and output vector. This assumption is unfortunately unrealistic in many applications.

Due to unmodeled dynamics and quantization errors, the residual error is highly dependent on the signals in the regressor. In recent years, there has been a significant effort to reformulate the identification problem to account for the existence of unmodeled dynamics as well as measurement noise (e.g., [8], [15], [16]). In this paper, we used simple measures of uncertainty described in the following. The parametric error measures are related to those of [15], which are based on the assumption of a bounded, but otherwise arbitrary, noise sequence added to (9). However, the measures described here do not make any assumption about the type or amount of noise
present. On the other hand, the interpretation of the uncertainty measures is somewhat subjective.

1) Residual Error: By expanding (11), the residual error can be rewritten as a quadratic function of the parameter vector $K^*$:

$$R_e = R_y - 2R_w^T K + K^T R_w K$$  \hspace{1cm} (18)

$$R_y = \sum y^T [n] y[n]$$  \hspace{1cm} (19)

$$R_w = \sum W[n] W^T [n]$$  \hspace{1cm} (20)

$$R_{wy} = \sum W[n] y[n].$$  \hspace{1cm} (21)

By using this notation, the least-squares solution and the corresponding residual error can be rewritten as

$$K^* = \frac{R_w^{-1} R_{wy}}{R_y}$$  \hspace{1cm} (22)

$$R_e^* = R_y - R_w^T R_w^{-1} R_{wy}.$$  \hspace{1cm} (23)

The residual error indicates how well the measured output $y[n]$ fits the linear model $W^T [n] K^*$. A residual error of zero would indicate a perfect match for every point $n$. Because of noise, quantization errors, and unmodeled dynamics, the actual residual error is greater than zero. If all the parameters were set to zero, the residual error would be equal to $R_w$. Since $R^*_e$ is the minimum residual error, it must be less than or equal to $R_w$. For an indication of how well the least-squares solution fits the data, a residual error index $E_I$ is defined by

$$E_I = \sqrt{\frac{R^*_e}{R_w}}.$$  \hspace{1cm} (24)

This index can be viewed as a percentage of how well the data fits the linear relationship.

2) Parametric Error: The residual error index indicates how well the data fits the model, but gives no indication as to how well the parameters are determined from the data. To investigate this property, we look again at the residual error. A graph of the residual error for the one-dimensional case is given in Fig. 2. The minimum residual error is the least-squares solution. The derivative of $R_e$ with respect to $K$ is zero at $K^*$. Therefore, we cannot look at the derivative for a measure of sensitivity. Consider then variations in the parameters which cause the residual error to double. The residual error due to a variation of $\delta K$ in the parameters is shown in (25). Setting $R_e$ equal to $2R^*_e$ yields (26):

$$R_e = R^*_e + \delta K^T R_w \delta K$$  \hspace{1cm} (25)

$$R_e^* = \delta K^T R_w \delta K.$$  \hspace{1cm} (26)

In the one-parameter case, this yields an estimate of the parametric sensitivity

$$\delta K = \frac{R^*_e}{R_w}.$$  \hspace{1cm} (27)

In the two-parameter case, the parabola of Fig. 2 becomes a paraboloid and its intersection with $R_e = 2R^*_e$ is an ellipse, as shown in Fig. 3. We now have two parametric errors $\delta K_1$ and $\delta K_2$, and several possible definitions. Three interesting values for the $\delta K_i$'s are examined. The first is the value of $\delta K_i$ that causes the entire increase in residual error, all other $\delta K_i$'s being zero. This corresponds to the intercept $\delta K_i^{(1)}$ of (26) with the $i$th axis. The second $\delta K_i^{(2)}$ is the maximum value each $\delta K_i$ can reach while satisfying (26). The third $\delta K_i^{(3)}$ is the component of the $\delta K_i$ of maximum norm. The formulas for determining the $\delta K_i$'s are

$$\delta K_i^{(1)} = \frac{R^*_e}{(R_w)_{ii}}$$  \hspace{1cm} (28)

$$\delta K_i^{(2)} = \sqrt{R^*_e (R_w^{-1})_{ii}}$$  \hspace{1cm} (29)

$$\delta K_i^{(3)} = \frac{R^*_e}{\lambda_{\text{min}}(R_w)^{1/2}}.$$  \hspace{1cm} (30)

A derivation of these equations is provided in the Appendix. $R^*_e$ and $R_w$ are defined in (23), (20). $(R_w)_{ii}$ is the $i$th diagonal element of $R_w$. $\lambda_{\text{min}}(R_w)$ denotes the minimum eigenvalue of $R_w$ (because of the definition of $R_w$, all its eigenvalues are real and non-negative). $x_{\text{min},i}$ is the $i$th component of the eigenvector (of norm 1) associated with the minimum eigenvalue. These definitions for the parametric errors hold for the $N$-dimensional case as well as the two-parameter case.

For each parametric error, we define a parametric error index

$$PE_i = \frac{\delta K_i}{K_i^*}.$$  \hspace{1cm} (31)

This index is equal to the percentage change in the parameter corresponding to a doubling of the residual error and provides an indication of how sensitive the residual error is to each
parameter estimate. Therefore, $PE_i$ provides a measure of the order of magnitude of the errors on the parameters. Fig. 4 depicts two cases with a parametric error index of 0.1 and 1. For $PE_i = 1$, only a slight increase in the residual error is caused by a large variation in the parameter estimate. Therefore, one would have a low degree of confidence in that particular estimate. On the other hand, for $PE_i = 0.1$, the residual error is highly sensitive to the parameter and more confidence can be placed in the estimate. We experimented with all three parametric error indexes. The second index is the most conservative and was preferred over the others.

The parametric error indexes were chosen to correspond to a doubling of the residual error. This is an arbitrary and also somewhat conservative choice: it means that we look for parametric variation that would lead to an increase of error equal to 100% of the minimum residual error. If another level had been chosen, all the parametric error indexes would have been scaled by the same constant factor. The parametric error indexes are especially useful to compare relative sensitivities of different parameters and to give "ballpark" values of the errors to be anticipated. An alternative is to assume that (9) is a perturbed noise of known magnitude. Assuming a known level of noise (as in [15]) is equivalent to fixing a certain level of residual error from which bounds on parametric errors can be deducted.

3) Persistency of Excitation: The parametric errors are directly related to the presence or absence of excitation, i.e., the identifiability of the parameters in ideal conditions.

Specifically, assuming that no noise or unmodeled effects were present, the convergence of the least-squares estimates to the true values is guaranteed if the matrix $R_m$ is nonsingular (a condition weaker, but close to the persistency of excitation condition, cf. [9]). If the condition is not satisfied, it can indeed be observed that some of the parametric error indexes will be infinite. Presence or absence of adequate excitation can, therefore, be assessed by the condition number of the matrix $R_m$ or by its minimum eigenvalue.

In the presence of noise or unmodeled dynamics, one must be careful, because the parametric errors could be large even if the matrix $R_m$ is well-conditioned. This occurs if there is a large amount of noise or modeling errors. The parametric error index (29) reflects this fact, being based on the product of the residual error with the elements of the inverse of the matrix $R_m$. Therefore, this error index will warn the user of a poor determination of a parameter, due to a poor matching of the data with the model as well as due to insufficient excitation. It turns out that this estimate of the parametric error is almost identical to the stochastic estimate found in [12], that is, modulo a factor $1/(N - n)$ present in the stochastic case, where $N$ is the number of data points and $n$ is the number of parameters. In the stochastic analysis, the parametric errors go to zero as the number of data points go to infinity, because the noise sequence is uncorrelated with the regressor signals. In practice, infinite precision is never reached on any parameter because of the errors in the model and the presence of correlated noise. The parametric error indexes also do not converge to zero as the number of data points becomes large.

IV. EXPERIMENTAL SETUP

The experimental setup used for this project is a slight modification of an industrial product of Aerotech, Inc. A block diagram of the overall system is shown in Fig. 5. Programs are down-loaded from an IBM PC to a development system for a digital signal processor of Motorola (DSP56001). Signals are commanded and measured via the power electronics/interface board to and from the stepper motor. A brief description is given for each of the subsystems.

1) Motor/Encoder: The motor is a two-phase permanent magnet stepper motor with 50 rotor teeth, giving a step angle of 1.8 deg. A 2000 line optical encoder is used to measure the rotor position. At 2000 counts per revolution, the encoder has a resolution of 0.18 deg, or 1/10 of the step angle.

2) Power Electronics: The commanded voltages are supplied by a 20-kHz pulsewidth modulated (PWM) amplifier. The maximum output voltage of the PWM is $\pm 80$ V. The maximum current for the motor is 6 A and the limit is enforced electronically. Due to the high-frequency noise caused by the PWM, a large sampling frequency was chosen to avoid aliasing. The major frequency component of the PWM noise is located around 20 kHz. A sampling frequency of 50 kHz was then selected. This corresponds to a sampling period of 0.02 ms.

3) Interface Board: An interface board serves as an input/output interface between the DSP board and motor. Two 12-bit D/A converters are used for commanding the phase voltages. Four 8-bit A/D converters are used for measuring the phase voltages and currents. Due to the maximum output voltage of the PWM, the D/A and A/D converters for the voltages are scaled to command and measure $\pm 80$ V. The resulting quantization errors are 0.04 and 0.625 V, respectively. Since the maximum current for the PM stepper motor is 6 A, the current A/D converters are scaled to $\pm 6$ A. This results in a quantization error of 0.047 A.

4) Motorola DSP56001 ADS: The DSP56001 Application Development System (ADS) is a tool to design and execute
data intensive DSP applications. The system contains a computer interface board which provides communication between the application development module (ADM) board and the user interface program located on the host computer. An assembler and linker are included for creating executable programs. These programs are then downloaded by the interface program/board to the ADM for execution.

5) PC and Software: Assembly level programs were written to control the operation of the motor and to record the data. The programs were created using the ASM56000 compiler and linker. The programs were then downloaded to the DSP board. The data collected by the assembly program/DSP board were stored in files with the DSP extension and then converted into the format of the software package MATLAB, where further processing was performed. Simulations were carried out using the nonlinear simulation package SIMNON. A complete model was developed that simulates the system using the ideal model (2), time delays, encoder, D/A and A/D converters, and current saturation.

A. Data Processing — Remarks

A few words are first in order about the measurement delays. Since the number of rotor teeth is 50, a mechanical speed of 3000 rpm corresponds to an electrical frequency of 2.5 kHz. Even if one samples at 50 kHz, a delay of one sampling period corresponds to a phase lag of 18 deg, which is significant enough to require consideration. A phase shift in the measurement of the phase currents leads to a rotation by the same angle in the DQ space. Since the electrical torque is proportional to \(i_q\) only, such a shift could significantly affect the torque. For this reason, the delays present in the system had to be precisely accounted for [2].

Due to the zero-order holds, the D/A converters create a delay between the commanded and actual voltages equal to half the sampling period. A delay is also caused by the time required to measure the state variables, calculate the DQ transformation, and send the new commanded voltages. The PWM circuit also contains a low-pass filter to reduce circulation in the voltage regulator of the high-frequency noise. The filter has a dc gain of 1 and a cut-off frequency of 10 kHz. The filter, PWM, and zero-order hold delays were combined into a single delay of approximately 1.67T. The commanded voltages have to be delayed by this amount to reflect the voltages effectively applied to the motor.

Another issue of significance in the experiments is the filtering of the noise before processing by the least-squares algorithms. In MATLAB, the data was transformed into the DQ coordinate system. Next the states were filtered in order to remove as much of the measurement (PWM) noise as possible. A third-order Butterworth filter was used with a cutoff frequency of 500 Hz for the D/Q currents and 100 Hz for the D/Q voltages.

This filtering was performed twice, forward and backward, to avoid the introduction of delays (using the MATLAB function filfilt). To obtain all the necessary signals for the identification procedure, the derivatives were reconstructed using the difference equation

\[
\dot{x}(k) = \frac{x(k) - x(k - 1)}{T},
\]

This reconstruction enhanced high-frequency noise. Therefore, after reconstruction, the derivatives were filtered again using a third-order Butterworth with a cut-off frequency of 500 Hz.

V. EXPERIMENTAL RESULTS

A. RL Measurements

Prior to the experiments, a few simple tests were performed to obtain estimates of the resistances and inductances of the phases. The phase resistances were first measured with an ohmmeter across each phase, giving a value of 0.6Ω for both phases. A set of experiments was then performed to measure the rotor step response at standstill. The rotor was first aligned with a phase by commanding a voltage to that phase. Once the rotor was set, a voltage step response was performed in the same phase. Because the rotor remained fixed (\(\omega = 0\)), the energized phase behaved like a simple RL circuit.

Table II gives the results of the experiment. The value of \(R\) is calculated by dividing the phase voltage by the steady-state current. The time constant \(\tau\) is equal to the time required for the current to reach 63 % of its maximum value. The value of the inductance is calculated by dividing \(\tau\) by the value of \(R\) determined above. Several different voltages were applied to each phase to test the dependence of \(L\) on current. The results show that, as the current increases, the inductance decreases, as expected due to the saturation effects. These effects were found to be moderate, however, so that an average value of \(L = 2.4\) mH could be assumed. The average resistance in both phases was within 21% of the ohmmeter measurements. In later experiments, the value of 0.6Ω was used whenever \(R\) was taken as known.

B. Simulated Data

Simulations of an acceleration experiment were performed to test the accuracy of the identification algorithms and the usefulness of the error indexes. Indeed, with simulations, the nominal ("true") parameters are known. Yet, because the actual quantization levels are included in the simulation using the complete model and because derivatives have to
be reconstructed, the identification is not perfect, even with simulated data.
An acceleration experiment was simulated using assumed values for the parameters. Both the one-stage and two-stage identification procedures were performed over the entire interval. Note that the interval was short (0.08 s), yet sufficient to determine all the parameters. Table III shows the identification results, including the estimates, the error index, the parametric error indexes, and the actual parametric errors, for both the one-stage and two-stage identification algorithms. The error index is about 8%. This error quantifies the matching of the model to the data. With simulated data, this error is due to quantization and sampling effects (reconstruction of the derivatives). It gives an indication of a bound on what to expect from the experiments. The parameter estimates are very close to the true values. The parametric error indexes turn out to be conservative measures of the actual errors (by about an order of magnitude), but correctly represent the relative distribution of the errors among the parameters. The two-stage identification gives more realistic values for the parametric error indexes. This is attributed to the large difference in the residual error between the electrical and mechanical equations (due to different units), leading to very conservative estimates of parametric uncertainty. Rescaling the variables would probably have achieved the desired result, but the two-stage procedure made it unnecessary. A simulation was then performed, using the estimated values from Table III. Fig. 6 shows the comparison of the angular velocity for the original and estimated parameter values. The estimated parameter values produced responses very close to the original responses.

C. Experimental Data
A 0.08-s acceleration experiment was performed to test the identification algorithms on actual data. The input voltages were chosen such that the phase currents would remain within limits and the motor would rapidly accelerate. The identification was carried out over the initial acceleration period of 0.01–0.03 s.

1) One-Stage and Two-Stage Identification Comparison: One- and two-stage identifications were performed on the data to compare the parameter estimates and the parametric error indexes. The identification results are shown in Table IV.

As with the simulation results, the parameter estimates for both methods are basically the same. For the first set of parameters \((R, L, K_m)\), the parametric error indexes are practically the same. The second set of parameters \((J, B, C)\) have significantly different parametric error indexes, consistently with what was observed in simulations. The residual error indexes \(E_I\) for the actual and for the simulated data are close (12 and 8%, respectively, for the one-stage procedure). The slight increase was to be expected, considering the noise caused by the PWM and due to other unmodeled effects. Overall, the results are very similar to those obtained with simulated data. As seen in the results the estimate of \(R\) was very imprecise. The parametric error index does indeed indicate a small sensitivity of the data to this parameter. The reason for this result is that \(R\) is very small (0.6Ω) and has a marginal effect for speeds greater than 40 rad/s (400 rpm). On the other hand, the inductance \(L\) dominates and the estimate is very close to the value obtained independently using the fixed-rotor step response. This result gives a confirmation of the validity of the identification procedure.

2) Known and Unknown Parameter Comparison: A series of two-stage identifications were performed to study the effect known parameters have on the other estimates. First, all the parameters were estimated. Then, the parameters which had very large parametric error indexes were each set to either a known value, or to zero. Table V shows the results for the parameters \(R, B,\) and \(C\). Note that the value of \(R\) has a
TABLE V
KNOWN AND UNKNOWN PARAMETER COMPARISON

<table>
<thead>
<tr>
<th>Parameter</th>
<th>None</th>
<th>R</th>
<th>R and θ</th>
<th>R and C</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.2698</td>
<td>(0.6)</td>
<td>(0.6)</td>
<td>(0.6)</td>
</tr>
<tr>
<td>J</td>
<td>0.0027</td>
<td>0.00281</td>
<td>0.00281</td>
<td>0.00281</td>
</tr>
<tr>
<td>Km</td>
<td>0.515</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>E_r = 12%</td>
<td>E_r = 13%</td>
<td>E_r = 13%</td>
<td>E_r = 13%</td>
<td></td>
</tr>
</tbody>
</table>

| J         | 0.000187 | 0.000182 | 0.000183 | 0.000183 |
| B         | 0.0032 | 0.0031 | (0) | 0.006 |
| C         | 0.0093 | 0.0672 | 0.118 | (0) |
| E_r = 10% | E_r = 10% | E_r = 11% | E_r = 12% |

---

Fig. 7. Actual and estimated \( i_q \) for estimated parameters of acceleration experiment.

Fig. 8. Actual and estimated \( \omega \) for acceleration experiment.

Fig. 9. Actual and simulated \( \omega \) for estimated parameters of deceleration experiment.

TABLE VI
ACCELERATION AND DECELERATION IDENTIFICATION COMPARISON

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Acceleration Estimate</th>
<th>Deceleration Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>(0.6) 0%</td>
<td>(0.6) 0%</td>
</tr>
<tr>
<td>J</td>
<td>0.00281 46.3%</td>
<td>0.00254 20.9%</td>
</tr>
<tr>
<td>Km</td>
<td>0.5 18.6%</td>
<td>0.4 12.5%</td>
</tr>
<tr>
<td>E_r = 13%</td>
<td>E_r = 10%</td>
<td></td>
</tr>
</tbody>
</table>

| J         | 0.000182 13.2%        | 0.000184 50.5%        |
| B         | 0.0031 214%           | (0) 0%                |
| C         | 0.0672 191%           | 0.169 105%            |
| E_r = 10% | E_r = 27%             |

reconstructed from the encoder position measurements, and estimated "means" obtained from a simulation using the estimated parameters. Matching of the simulated and experimental data is very good, especially over the interval [0.01, 0.03] for which the identification was performed. Quite expectedly, however, errors accumulate over the period of the experiment.

D. Deceleration Experiment

A deceleration experiment was performed to test the reproducibility of the results. The identification was carried out over the entire deceleration ([0.02, 0.08] seconds). A comparison of the acceleration and deceleration results is shown in Table VI. As in the acceleration experiment, the value for \( R \) was very imprecise and was then set to its predetermined value of 0.6Ω. In addition, the identification procedure attempted to set \( B \) to a negative value and was, therefore, set to 0. The values of \( L \) and \( J \) determined by the acceleration and the deceleration experiments are very close. The residual error for the mechanical variables is larger for the deceleration experiment. This less accurate matching is reflected in an increased uncertainty of the parameter \( J \).

A simulation of the deceleration experiment was performed using the parameter values from Table VI. Fig. 9 shows the actual and estimated \( \omega \). As indicated by the error index, the simulated deceleration has a less accurate match than the acceleration. The deceleration experiment has the drawback
of exhibiting nearly constant deceleration, possibly leading to difficulties in separating the inertial forces from the Coulomb friction (although the estimated inertia is consistent with the previous results). To improve the match, it would be necessary to further develop the model of the motor and perhaps change the speed profile. Further analysis was not pursued since the main parameters affecting control performance, namely $L, J,$ and $K_m$ were reliably estimated.

VI. CONCLUSIONS

A batch least-squares identification algorithm was used to estimate the parameters of a PM stepper motor. The simulation results showed that the estimates were very close to the true values and that the parametric error indexes gave useful (yet conservative) measures of the actual errors in the parameter estimates. When applied to actual data, the residual error index was of the same order of magnitude as that obtained with the simulated data, a confirmation of the validity of the model and procedure used. The results of the procedure were also validated by comparing its results with those of simple tests, such as the fixed-rotor step response, and by comparing acceleration and deceleration experiments.

For the high-speed experiments carried out here, the resistance had a negligible effect and it was preferable to estimate it separately from a test at standstill. The frictional terms were also difficult to estimate and could be neglected. This situation would be different if a load such as a ball-screw stage had been connected to the motor. The main parameters $L, J$ and $K_m$ affecting control performance were reliably estimated in short experiments (less than a tenth of a second). Further, the simulations gave results that matched closely those observed in the experiments.

The simplified model used for identification purposes is based on several assumptions that are only approximately valid. The fixed-rotor step responses indicated that the inductance depends on the current. This effect, along with the other unmodeled dynamics, can cause errors in the parameter estimates. An additional error is created by the quantization of the measurements. The most significant contribution of the quantization errors occurs during the reconstruction of the derivatives. The power electronics also contribute a significant amount of PWM noise. In addition to unmodeled dynamics and noise, heating contributes to parameter fluctuations over a period of time. It would be futile to try to model some of these effects, but others could be included and could lead to a more accurate modeling. As long as a form linear in the parameters is obtained, the methods of this paper can still be used.

The identification method is simple enough that it can easily be made recursive and implemented in real-time (a full implementation of the algorithm on the DSP56001 was carried out in [4]). The procedure is able to effectively estimate the parameters within a very short time (a tenth of a second). Therefore, it can be applied to real-time self-tuning and adaptive state-space control at high speed. An advantage over traditional methods is that no special configuration of the motor is necessary, and responses from the normal operation can be used. Additional effects, such as detent torque can also be added to the model and identified for precise control. Since the experiments were obtained on an industrial setup, the results have a clear potential application to industrial processes for improved performance and efficiency.

APPENDIX

We derive (28)–(30), starting from (18). Letting $K = K^* + \delta K$ and using (22), (23), one finds that

$$R_w = R_w^* + \delta K^T R_w \delta K.$$  \hspace{1cm} (33)

Choosing $R_w = 2R_w^*$ leads to the equation specifying $\delta K$, i.e.:

$$R_w^* = \delta K^T R_w \delta K.$$  \hspace{1cm} (34)

The first error index (28) corresponds to setting all the components of $\delta K$ to zero, except for the $i$th component, so that

$$R_w^* = (\delta K_i)^2 (R_w)_i.$$  \hspace{1cm} (35)

The first error index follows directly from (35). Now, the second error index requires finding the $\delta K$ that maximizes $\delta K^T$, subject to (34). Let $e_i$ be the column vector whose components are all zero, except the $i$th component which is equal to 1. Using a Lagrange multiplier approach, we set the derivative of $(e_i^T \delta K)^2 + \lambda (R_w^* - \delta K^T R_w \delta K)$ equal to zero, so that

$$e_i^T e_i \delta K - \lambda R_w \delta K = 0.$$  \hspace{1cm} (36)

Multiplying (36) on the left by $\delta K^T$ and using the constraint (34) gives

$$(e_i^T \delta K)^2 = \lambda R_w.$$  \hspace{1cm} (37)

On the other hand, multiplying (36) on the left by $e_i^T R_w^{-1}$ gives

$$e_i^T R_w^{-1} e_i = \lambda.$$  \hspace{1cm} (38)

The second parametric error index is then obtained by combining (37) and (38). For the third parametric error index, one uses the fact that

$$\delta K^T R_w \delta K \geq \lambda_{\text{min}}(R_w) \lVert \delta K \rVert^2$$  \hspace{1cm} (39)

with the equality obtained when $\delta K$ is a multiple of the eigenvector associated with the minimum eigenvalue.

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Marc Bodson (M'86), for a photograph and biography, please see the March 1993 issue of Transactions on Control Systems Technology.

John Chiasson (S'82-M'84), for a photograph and biography, please see the March 1993 issue of Transactions on Control Systems Technology.