Impact of Load Forecast Uncertainty on LMP

Rui Bo, *Student Member, IEEE* and Fangxing Li, *Senior Member, IEEE*

***Abstract* — In day-ahead power markets, the calculation of locational marginal price (LMP) relies on the load forecasting results. It is well known that short-term load forecasting results always contain certain degree of errors mainly due to the random nature of the load. At the same time, LMP step change exists at critical load level (CLL). Therefore, it is interesting to investigate the impact of load forecast uncertainty on LMP. With the assumption of distribution of actual load, this paper formulates the probability mass function of the random variable LMP_t, LMP at time t, and then proposes the concept of probability-based expected LMP. Two useful curves, deterministic LMP versus forecasted load and expected LMP versus forecasted load, are presented. The first curve is designed to help identify the trustworthy regions of traditional LMP-Load curve. The second curve is demonstrated to be smooth and therefore eliminates the step changes in deterministic LMP simulation. The proposed concept and method are illustrated on a modified PJM 5-bus system.**

***Index Terms*—power markets, energy markets, locational marginal pricing (LMP), optimal power flow (OPF), critical load level, normal distribution, uncertainty.**

I. Introduction

PREDICTION of load, especially short-term load consumption, has long been an important topic in academia and industrial research and practices [1]. With the deregulation of power industry and the adoption of locational marginal pricing (LMP) methodology, LMP forecasting has attracted lots of attentions due to the significance of LMP in delivering market price signal and being used for settlement [2-4].

It is known that LMP is the shadow price of energy balance equation of the economic dispatch model, and can be decomposed into three components with each representing the marginal energy price, marginal loss price, and marginal congestion price, respectively [5-6]. The decisive factors of LMP include supply offers, demand bids, load forecasting, and system outage plan. In a day-ahead power market, once the market is closed (normally at 12:00 noon before the operating day), the offers and bids are fixed and a transmission network model will be used in day-ahead marketing scheduling. Nevertheless, load remains uncertain because there is essentially no way to discover the exact load of each hour of the next operating day. Load forecasting is applied to address this issue, but performance varies with models, algorithms and the nature of the problem. It is apparent that the uncertainty of load directly leads to the uncertainty of LMP. Therefore, like analysis of other economic impact of load forecasting [7-9], it is necessary to investigate how the LMP will be affected by uncertainty of load, or, uncertainty of load forecasting results in practice.

A methodology of computing LMP sensitivities with respect to load in AC Optimal Power Flow (ACOPF) framework has been presented in [10]. Approaches for DC Optimal Power Flow (DCOPF) have been proposed in [11-12]. In [12], a perturbation method is applied to identify the next critical load level (CLL), defined as a load level at which a LMP step change occurs, and the corresponding new binding limit and new unbinding limit. This method is demonstrated to be very computationally efficient since it does not require multiple optimization runs. In addition, it enables the computationally efficient study of LMP on any range of load variations.

Figure 1 shows a typical LMP versus load curve [11-12] for a sample system slightly modified from the original PJM 5-bus system defined in [13]. Losses are ignored in those studies such that this research is concentrated on the overall behavior of LMP due to congestions. In theory, the horizontal axis denotes the actual load. In practice, the axis represents the forecasted load, since forecasted load is used to perform dispatch and LMP calculation.

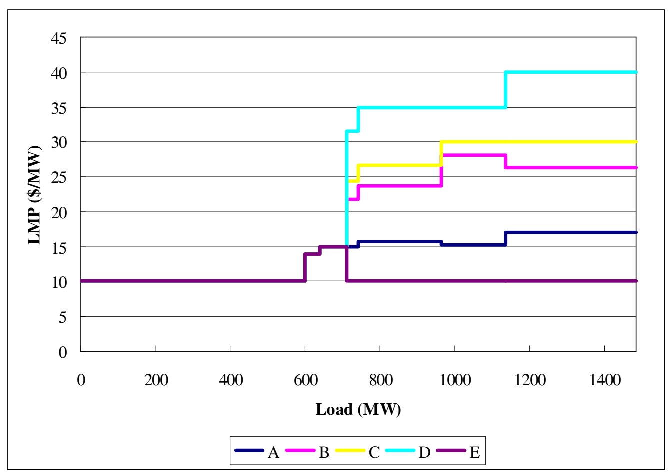

Fig. 1. LMP at all buses with respect to different system loads for the modified PJM 5-bus system.

It can be seen that there is a step change of LMP when load increases to a CLL, e.g., the load level at 600 MW, 640MW, 711.81MW, etc). At each CLL, a new binding limit, either a transmission line thermal limit or a generator capacity limit, occurs. Meanwhile, there is a change of the marginal unit set and marginal generation sensitivity with respect to load, which essentially leads to the LMP step change.

The sensitivity of LMP with respect to load is highly sensitive at CLLs, and mathematically is evaluated as infinite. Slight difference in forecasted load may result in dramatic difference in LMP. For example, LMP at bus D is

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\$15.00/MWh when forecasted load is 711.80MW whereas the price soars to \$31.46/MWh when forecasted load is slightly off by 0.1MW, making it 711.90MW. On the other hand, it is very likely for a well-tuned load forecasting tool to produce a result with a bigger error than 0.1MW for a target load at 700MW level. Therefore, load forecast error may significantly affect LMP and the corresponding payment and revenue of market participants. In addition, the step change characteristic of LMP leads to the ambiguity of LMP evaluation at CLLs. For example, when forecasted load happens to be 711.81MW, the LMP at bus D may be one of the two values: \$15.00/MW or \$31.46/MW. But it is not well justified which one should be taken. Taking into account the load variation direction (namely, increase or decrease) may be an option.

There are a few reasons leading to the uncertainty of load forecasting result. First of all, future load is a random variable and cannot be predicted accurately. Different load forecasting method may excel in different applications, but no one can achieve 100% accuracy. There is always a certain error range associated with the forecasted results. Second, every method has its own theoretical foundations and likely produce results differently than others. Third, even for the same method, the results may be different by using different settings and tunings. Last, most methods suffer from missing data and highly rely on the input data accuracy.

Although the uncertainty in load forecast is unavoidable, the actual load or forecast error does follow certain probability distribution, which enables the study of the correlation between load forecast and LMP in a probabilistic sense. This is the motivation of this research work.

This paper is organized as follows. Section II introduces the assumptions upon which the research relies and formulates the probability mass function of LMP. Section III presents the formulation for calculating expected LMP. Section IV presents an example based on the PJM 5-bus system. Section V summarizes the paper and points out future works.

II. Probability Mass Function of LMP

A. Assumptions

Actual load, or load forecasting error can be assumed to be a random variable and follow certain probability distribution. Normal distribution is frequently used and will be used in this model to describe the actual load at hour *t*. Then, we have

$$D_t \sim N(\mu_t, \sigma_t^2) \quad (1)$$

$$\varphi(x) = \frac{1}{\sigma_t \sqrt{2\pi}} e^{-\frac{(x-\mu_t)^2}{2\sigma_t^2}} \quad (2)$$

$$\Phi(x) = \int_{-\infty}^{x} \varphi(u)du \quad (3)$$

where

D_t = actual load at hour *t* which is a random variable;

N denotes for normal distribution function;

μ_t = mean of D_t;

σ_t^2 = variance of D_t;

$\varphi(x)$ = probability density function of D_t;

$\Phi(x)$ = cumulative density function of D_t.

It should be noted that μ_t may or may not be equal to the forecasted load at hour *t*, D_t^F.

B. Models for the LMP-Load Curve

Since the random variable D_t is represented by the real axis, the LMP-Load curve (like the one shown in Fig. 1) will be extended correspondingly, as shown in Fig. 2, to facilitate the following study.

As shown in Fig. 2, The load is divided into *n-1* segments by load level sequence $\{D_i\}_{i=1}^n$. D_1 represents load level with zero load, and D_n represents the maximum load the system can afford due to limit of total generation resources and transmission capability. Associated with each load segment *i*, there is a corresponding actual LMP value, p_i, which is considered a constant.

The extended LMP-Load curve includes two extra segments. One is for load from D_0 to D_1, where D_0 denotes negative infinite load, and the associated price is zero, as denoted by p_0. The second additional segment is defined as the load range from D_n to D_{n+1}, where D_{n+1} represents positive infinite load. In this segment the price is set to be p_{n-1}, same as the price in *(n-1)*th segment if we ignore the demand response. The reason is that, for any load greater than D_n, it will be curtailed to within the system maximum delivery capability, namely, D_n.

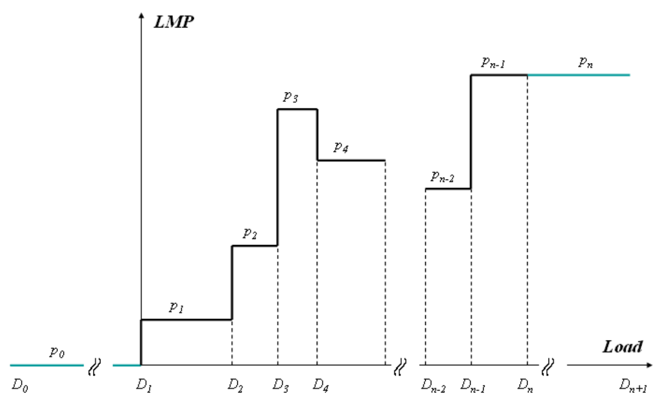

Fig. 2. Extended LMP versus Load Curve.

The curve can be formulated as

$$LMP(D)=\begin{cases} p_0 & ,D_0<D\le D_1 \\ p_1 & ,D_1<D\le D_2 \\ p_2 & ,D_2<D\le D_3 \\ \vdots & \\ p_{n-1} & ,D_{n-1}<D\le D_n \\ p_n & ,D_n<D\le D_{n+1} \end{cases} \tag{4}$$

Where

$p_0=0$

$p_n=p_{n-1}$

$D_0=-\infty$

$D_1=0$

$D_{n+1}=\infty$

The compact representation is given as follows:

$$LMP(D)=\{p_i \mid i\in\{0,1,\cdots,n\}, D_i<D\le D_{i+1}\} \tag{5}$$

Apparently, $\frac{\partial LMP}{\partial D}$, the LMP sensitivity with respect to load, is infinite at load level sequence $\{D_i\}_{i=1}^{n-1}$, which are the CLLs.

C. Probability Mass Function of LMP

LMP is a function of load and D_t, load at hour *t*, which is a random variable. Therefore, LMP at hour *t*, LMP_t, is a function of D_t. Figure 3 shows the LMP-Load curve and the probability distribution of D_t. It can be seen from Fig. 3 that LMP_t is a discrete random variable, with *n+1* possible values as denoted by the sequence $\{p_i\}_{i=0}^{n}$. It is true that there may be identical values in the sequence $\{p_i\}_{i=0}^{n}$.

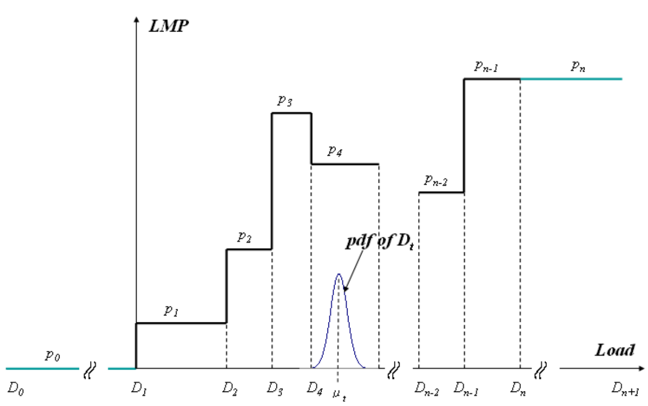

Fig. 3. LMP-Load curve and probability distribution of D_t.

Furthermore, the probability that LMP_t has a value of p_i may be formulated as

$$\begin{aligned}\Pr(LMP_t=p_i)&=\int_{D_i}^{D_{i+1}}\varphi(u)du \\ &=\Phi(D_{i+1})-\Phi(D_i)\end{aligned} \tag{6}$$

The cumulative density function $\Phi(x)$ can also be well estimated by available approximation methods [15]. The schematic graph of the probability mass function of LMP_t is shown in Fig. 4. It should be pointed out that normally the graph is presented in such a way that possible values are sorted in ascending order, and identical values are merged together. However, it is not done in Fig. 4 just for the purpose of better illustration.

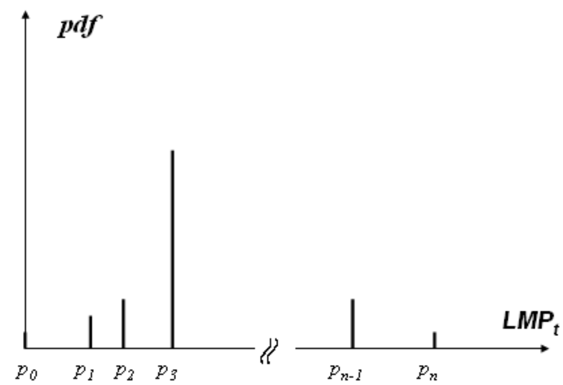

Fig. 4. Probability Mass Function of LMP at hour *t*.

D. Probability of Deterministic LMP versus Forecasted Load Curve

At hour *t*, the forecasted load is D_t^F and the associated LMP can be identified by looking up the LMP-Load curve as shown in Fig. 2. Suppose the corresponding LMP is p_j, namely,

$$LMP(D_t^F)=p_j,\qquad D_j<D_t^F\le D_{j+1} \tag{7}$$

where

$LMP(D_t^F)$ is the LMP corresponding to the forecasted load D_t^F. It may be called predicted LMP or deterministic LMP.

Then, the probability of having $LMP(D_t^F)$ as the price is obtained from the probability mass function as shown in Fig. 4. Hence, we have

$$\begin{aligned}\Pr(LMP_t=LMP(D_t^F))&=\int_{D_j}^{D_{j+1}}\varphi(u)du \\ &=\Phi(D_{j+1})-\Phi(D_j)\end{aligned} \tag{8}$$

When the above equation is evaluated for every D_t^F in the interval [D_1 , D_n], a probability versus D_t^F curve will be readily available. Each point of the curve represents the probability of having $LMP(D_t^F)$ as the price when load is D_t^F . When combined with the LMP-Load curve, the LMP probability versus Load curve delivers very useful information such as how likely the LMP corresponding to the actual load, namely, LMP_t , will coincide with the deterministic LMP, namely, $LMP(D_t^F)$.

It would also be interesting to study the probability of having price in the neighborhood of $LMP(D_t^F)$, namely, $\Pr(MP(D_t^F)*(1-\alpha\%) \le LMP_t \le LMP(D_t^F)*(1+\alpha\%))$, where α is the tolerance percentage.

This will be elaborated in Section IV that demonstrates the above discussion with a numeric study.

III. Expected LMP

A. Expected LMP

Since LMP_t is a random variable, it would be interesting to see the expected value of LMP at hour *t*.

$$\begin{aligned} E(LMP_t) &= \sum_{i=0}^{n} \Pr(LMP_t = p_i) * p_i \\ &= \sum_{i=0}^{n} (\int_{D_i}^{D_{i+1}} \varphi(u)du) * p_i \qquad (9) \\ &= \sum_{i=0}^{n} p_i * \int_{D_i}^{D_{i+1}} \frac{1}{\sigma_t \sqrt{2\pi}} e^{-\frac{(u-\mu_t)^2}{2\sigma_t^2}} du \end{aligned}$$

It can be seen that $E(LMP_t)$ is a function of μ_t and σ_t . The function is defined as

$$f(\mu_t, \sigma_t) \overset{\Delta}{=} E(LMP_t) \qquad (10)$$

B. Expected LMP versus Forecasted Load Curve

If $f(\mu_t, \sigma_t)$ is evaluated for every μ_t in the interval [D_1 , D_n], the expected LMP versus μ_t curve will be obtained. Furthermore, if it is assumed that the forecasted load D_t^F is always equal to μ_t , the curve will depict the expected LMP with respect to the forecasted load. This will be exemplified in the Numeric Study section next.

It should be noted that $f(\mu_t, \sigma_t)$ is continuously differentiable at μ_t . Therefore, the sensitivity of expected LMP with respect to μ_t is derived as

$$\begin{aligned} \frac{\partial f(\mu_t, \sigma_t)}{\partial \mu_t} &= \sum_{i=0}^{n} p_i * \int_{D_i}^{D_{i+1}} (\frac{1}{\sigma_t \sqrt{2\pi}} e^{-\frac{(u-\mu_t)^2}{2\sigma_t^2}}) * \frac{2(u-\mu_t)}{2\sigma_t^2} * du \qquad (11) \\ &= \sum_{i=0}^{n} p_i * \int_{D_i}^{D_{i+1}} \frac{u-\mu_t}{\sigma_t^3 \sqrt{2\pi}} e^{-\frac{(u-\mu_t)^2}{2\sigma_t^2}} du \end{aligned}$$

In addition, it can be easily proved that $\frac{\partial f(\mu_t, \sigma_t)}{\partial \mu_t}$ has a finite upper bound.

IV. Numeric Study

In this section, numeric study will be performed on a PJM 5-Bus system [13] with slight modification. The modifications are for illustration purpose and are detailed in [12].

To calculate the LMP versus load curve as shown in Figure 1, it is assumed that the system load change is distributed to each nodal load proportional to its base case load for simplicity. Therefore, the load change is equally distributed at Buses B, C and D since each has 300 MW load in the base case. Fig. 5 shows the configuration of the system.

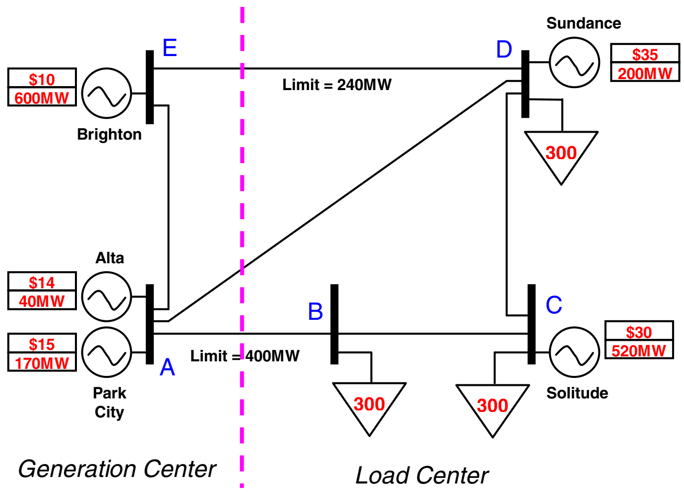

Fig. 5. The Base Case Modified from the PJM Five-Bus Example.

The critical load levels (CLLs) and the corresponding LMPs at each bus are shown in Table 1. These data are the data source of Fig. 1 and are calculated by the efficient solver presented in [12].

Table 1. CLL and LMPs.

CLL(MW)	*LMP@A*	*LMP@B*	*LMP@C*	*LMP@D*	*LMP@E*
0.00	10.00	10.00	10.00	10.00	10.00
600.00	14.00	14.00	14.00	14.00	14.00
640.00	15.00	15.00	15.00	15.00	15.00
711.81	15.00	21.74	24.33	31.46	10.00
742.80	15.83	23.68	26.70	35.00	10.00
963.94	15.24	28.18	30.00	35.00	10.00
1137.02	16.98	26.38	30.00	39.94	10.00
1484.06	16.98	26.38	30.00	39.94	10.00

For simplicity and better illustration, it is assumed that μ_t is always equal to forecasted load D_t^F , and the standard deviation σ_t is taken as 5% of the mean μ_t .

Probability mass function of LMP_t for bus D is calculated and shown in Table 2. It delivers the fact that the deterministic LMP may or may not be the price with the highest probability. For example, when forecasted load is 900MW, the corresponding deterministic LMP is $35/MW and has a high probability of 99.98%. However, the deterministic LMP $31.4571/MW for forecasted load 730MW has the second highest probability.

Table 2. PMF of LMP_t for bus D

LMP($/MW)	Probability when D^F_t=730MW	Probability when D^F_t =900MW
0	0	0
10	0.0002	0
14	0.0067	0
15	0.3023	0
31.4571	0.3280	0.0002
35	0.3629	0.9998
39.9427	0	0

The expected LMP for the above cases are compared with the deterministic LMP, $LMP(D_t^F)$, which are shown in Table 3. It shows that the expected LMP is almost identical to the deterministic LMP when forecasted load is 900MW while differs a bit when D_t^F is 730MW. These observations are consistent with the data shown in Table 2.

Table 3. Expected LMP and Deterministic LMP for bus D

D^F_t(MW)	Expected LMP($/MW)	Deterministic LMP($/MW)
730.0000	27.6485	31.4571
900.0000	34.9989	35.0000

Figure 6 shows the LMP probability at bus B versus forecasted load chart. Figure 7 shows the same chart with 10% price tolerance.

By comparing Figure 6 and Figure 1, we can see that the low probabilities occur near the critical load levels. When 10% price tolerance is considered, the probabilities for prices in the range of [90%, 110%] are lumped together. Therefore, as seen in Figure 6 and Figure 7, the probability at load 1137MW is about 0.54, while it increases to nearly 0.99 with 10% price tolerance considered because in this case the price difference at CLL 1137.02MW is within 10%.

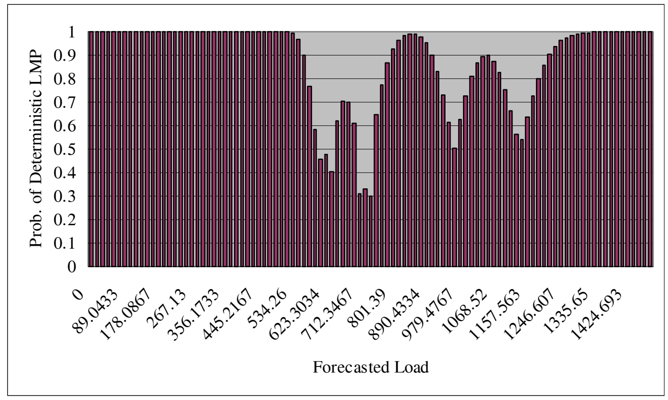

Fig. 6. Probability of deterministic LMP at bus B versus forecasted load.

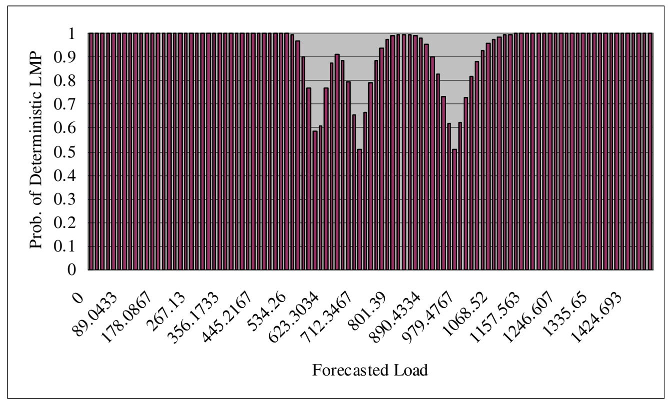

Fig. 7. Probability of deterministic LMP at bus B versus forecasted load (with 10% price tolerance).

The expected LMP versus forecasted load curve is shown in Figure 8.

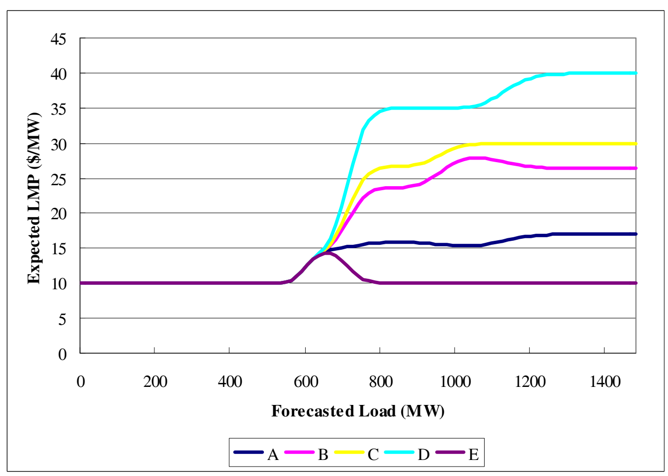

Fig. 8. expected LMP versus forecasted load.

Compared with the deterministic LMP-Load curve in Fig. 1, the probability-based expected LMP-Load curve is much smoother and no step change exists. This nice characteristic shows that if the price simulation is based on the probabilistic approach described in this paper, the error or uncertainty with respect to the actual LMP in operation will be reduced because of the elimination of step changes as shown in Fig. 8.

In other words, the proposed approach enables the probability-based expected LMP, among other approaches [11], to be used in price forecast to reduce price uncertainty caused by step changes.

V. Conclusions

Load forecasting uncertainty exists due to a variety of reasons, and is the major reason of the LMP uncertainty due to the step change characteristic of LMP-Load curve. This paper therefore studies the LMP uncertainty with respect to load in a probabilistic sense.

First, with the assumption of normal distribution of actual load, the probability mass function of LMP at hour t is computed. Second, probability of deterministic LMP versus forecasted load curve is formulated. This curve delivers the information of how likely the actual LMP will coincide with the deterministic LMP. Third, the expected LMP is derived and the expected LMP versus forecasted load curve is proposed. The sensitivity of the curve is derived and shown to be bounded by finite numbers. In addition, the expected LMP versus forecasted load curve is highly smooth and has no step changes.

The proposed concept and method are illustrated on a modified PJM 5-bus system. The results provide additional and useful information for understanding the LMP-Load curve from a probabilistic perspective.

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VII. Biographies

Rui Bo (S'02) received his B.S. and M.S. degrees both in electric power engineering from Southeast University of China in 2000 and 2003, respectively. From 2003 to 2005, he worked at ZTE Corporation and Shenzhen Cermate Inc., respectively. He started his Ph.D. study at The University of Tennessee, Knoxville, in January 2006. His current interests include power system operation and planning, power system economics and market simulation.

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