Aggregation of Multiple Induction Motors using MATLAB-based Software Package

Arif Karakas, Fangxing Li, *Senior Member, IEEE,* and Sarina Adhikari, *Student Member, IEEE*

Abstract — **This paper presents an aggregate model of multiple induction motors using a single equivalent circuit model, which simplifies the computation while simulating dynamic behavior of large power systems. The simulations and analysis are carried out using MATLAB-based software package. The performance of the aggregation model of multiple motors is verified by comparing the results obtained from the sum of individual induction motors and the aggregation model. The results obtained are satisfactory and reasonable.**

***Index Terms*— *Aggregation model, induction motors, Matlab simulation*.**

I. INTRODUCTION

LOAD model in large power systems has received increasing concerns in recent years for system operation and control [1-2]. Induction motors connected to power systems make up a major part of the total system loads and affect the system dynamics, especially the system stability. It has been recognized that high percentage of induction motor loads in the system prevents the normal voltage recovery following the fault [3]. However, it is not practical to model every individual induction motor in the simulation. Also, the modeling of a large number of individual induction motors during the simulation studies can be highly time-consuming. Therefore, some simplifications are necessary to represent a group of induction motors as a single equivalent unit which characterizes the aggregate model.

There are various aggregation methods proposed in different literatures [4-9]. The accuracy of the methods varies according to the assumptions used in obtaining the parameters of the aggregate model.

In [4], Abdel-Hakim and Berg proposed a method based on the steady-state theory of induction motor modeling. To obtain the parameters of the aggregation model, a single equivalent circuit is used by neglecting the stator resistance while meeting the power invariance condition. In [5], Franklin and Morelato developed an aggregation method using a fictitious impedance model based on steady-state theory, which avoided some common simplifications. In [6], Lem and Alley compared the third order and the fifth order aggregate induction motors and confirmed the need for a fifth order motor model as the size of motor increases. In [7], Pillay *et al.* proposed a new aggregate model based on the transformer-type equivalent circuit and used a grouping criterion to classify homogeneous motors.

The goal of this paper is to present an aggregate induction motor model using a single equivalent motor model, which aims to simplify the computation for the dynamic behavior simulation of large power systems. MATLAB-based software package including Simulink and SimPowerSystems is used in all simulations. The test results clearly demonstrate the validity and the accuracy of the proposed aggregation model.

II. THE AGGREGATION METHOD

Generally, an induction motor is represented by an equivalent circuit as shown in Fig. 1. As previously mentioned, there are several aggregation methods developed by researchers in the past. One of them is a method proposed by Kataoka *et al* in which the parameters of the equivalent circuit of the aggregate induction motor are determined for the case of two induction motors connected as power system loads [9]. The equivalent circuit parameters of the aggregation model are determined based on the same procedure and can be represented in Fig. 2 for N induction motors.

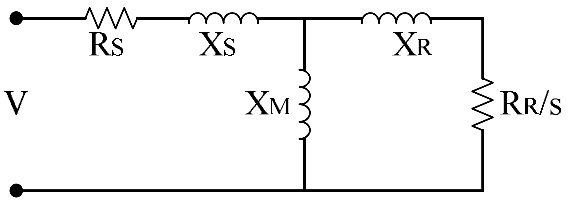

Fig. 1. Classical equivalent circuit model of an induction motor.

R_S	Stator resistance
X_S	Stator reactance
R_R	Rotor resistance
X_R	Rotor reactance
X_M	Magnetizing reactance
s	Slip of the induction motor

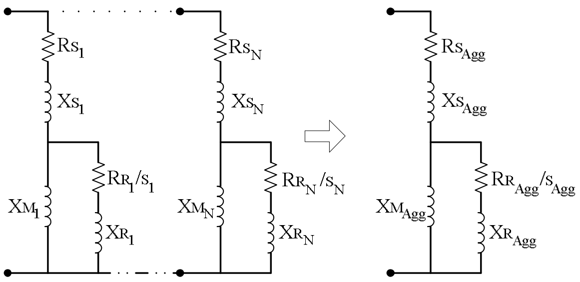

Fig. 2. Derivation of equivalent circuit parameters of the aggregate model.

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978-1-4244-3811-2/09/$25.00 ©2009 IEEE

For individual induction motors, it is assumed that all parameters of the equivalent circuit in Fig. 2 are known or can be obtained. The parameters of the aggregate induction motor are determined from two operating conditions: no-load and locked- rotor conditions.

In the no-load operating condition, it is assumed that the slips of all the induction motors are equal to zero, i.e., $s_1 = \ldots = s_N = s_{Agg} = 0$. Therefore, the no-load impedance of each individual motor is

$$Z_{NL_i} = R_{S_i} + j\cdot\left(X_{S_i} + X_{M_i}\right) \quad , \quad i=1, 2,..., N \qquad (1)$$

where N is the number of the induction motors.

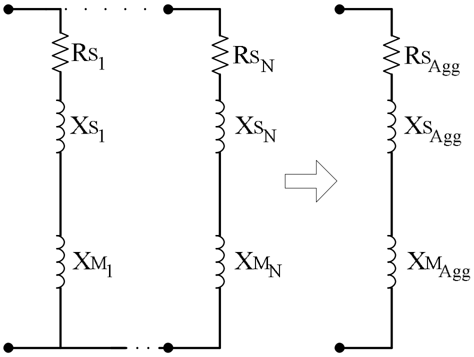

Fig. 3 No-load operating condition.

As commonly assumed, all N motors are connected in parallel at the same bus as shown in Fig. 3. Hence, the total no-load impedance is given by

$$Z_{NL_{eq}} = \frac{1}{\sum_{i=1}^{N} \frac{1}{Z_{NL_i}}} \qquad (2)$$

The no-load impedance of the aggregation model is given by

$$Z_{NL_{Agg}} = R_{S_{Agg}} + j\cdot\left(X_{S_{Agg}} + X_{M_{Agg}}\right) \qquad (3)$$

Comparing (2) and (3), we have

$$R_{S_{Agg}} = real\left\{Z_{NL_{eq}}\right\} \qquad (4)$$

$$\left(X_{S_{Agg}} + X_{M_{Agg}}\right) = imag\left\{Z_{NL_{eq}}\right\} \qquad (5)$$

In the locked-rotor operating condition, it is assumed that the slips of all the induction motors are equal to unity, i.e., $s_1 = \ldots = s_N = s_{Agg} = 1$. Therefore the locked-rotor impedance of each individual motor is given by

$$Z_{LR_i} = R_{S_i} + R_{R_i} + j\cdot\left(X_{S_i} + X_{R_i}\right) \quad , \quad i=1, 2,..., N \qquad (6)$$

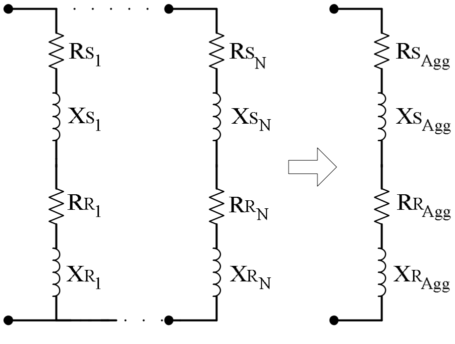

Fig. 4. Locked-rotor operating condition.

Again, as all N motors are connected in parallel at the same bus as shown in Fig. 4, the total locked-rotor impedance is given by

$$Z_{LR_{eq}} = \frac{1}{\sum_{i=1}^{N} \frac{1}{Z_{LR_i}}} \qquad (7)$$

The locked-rotor impedance of the aggregation model is given by

$$Z_{LR_{Agg}} = R_{S_{Agg}} + R_{R_{Agg}} + j\cdot\left(X_{S_{Agg}} + X_{R_{Agg}}\right) \qquad (8)$$

Comparing (7) and (8), we obtain

$$R_{S_{Agg}} + R_{R_{Agg}} = real\left\{Z_{LR_{eq}}\right\} \qquad (9)$$

$$\left(X_{S_{Agg}} + X_{R_{Agg}}\right) = imag\left\{Z_{LR_{eq}}\right\} \qquad (10)$$

$R_{R_{Agg}}$ can be obtained from (4) and (9). This is given by

$$R_{R_{Agg}} = real\left\{Z_{LR_{eq}}\right\} - real\left\{Z_{NL_{eq}}\right\} \qquad (11)$$

The reactance parameters ($X_{S_{agg}}$ and $X_{R_{Agg}}$) of the aggregation model can be determined based on the IEEE Standard 112-2004, which defines the ratio (n) of leakage inductances of induction motors based on the motor class [10]. This is given by

$$n = X_{S_{Agg}} \big/ X_{R_{Agg}} \qquad (12)$$

Combining Eqs. (10) and (12), we have

$$X_{S_{Agg}} = \frac{n}{n+1}\cdot imag\left\{Z_{LR_{eq}}\right\} \qquad (13)$$

$$X_{R_{Agg}} = \frac{1}{n+1}\cdot imag\left\{Z_{LR_{eq}}\right\} \qquad (14)$$

Hence, the magnetizing reactance $X_{M_{agg}}$ can be found from (5) and (13). This is given by

$$X_{M_{Agg}} = imag\left\{Z_{NL_{eq}}\right\} - \frac{n}{n+1}\cdot imag\left\{Z_{LR_{eq}}\right\} \qquad (15)$$

From Eqs. (1) through (15), all parameters of the aggregate model as shown in Fig. 2 have been obtained except the motor slip. To calculate the slip, it is assumed that each individual induction motor has a steady-state slip value of s_1, s_2, ..., and s_N, respectively. And the aggregate motor has a slip of s_{Agg}. Therefore, the impedance of each individual motor is given by

$$Z_i = Z_{S_i} + \frac{Z_{M_i}\cdot Z_{R_i}}{Z_{M_i} + Z_{R_i}} \quad , \quad i=1, 2,..., N \qquad (16)$$

where

$$Z_{S_i} = R_{S_i} + j\cdot X_{S_i}$$

$$Z_{M_i} = j\cdot X_{M_i}$$

$$Z_{R_i} = \frac{R_{R_i}}{s_i} + j\cdot X_{R_i}$$

Since all N motors are connected in parallel at the same bus as shown in Fig. 2, the total equivalent impedance can also be obtained by

$$Z_{eq} = \frac{1}{\sum_{i=1}^{N} \frac{1}{Z_i}} \qquad (17)$$

Considering the impedance of the aggregate motor $Z_{Agg} = Z_{eq}$ as shown in Fig. 2, the aggregate motor slip, s_{Agg}, can be formulated similar to [9] as follows:

$$\alpha \cdot s_{Agg}^2 + \beta \cdot s_{Agg} + \gamma = 0 \tag{18}$$

where

$$\alpha = \left(X_{M_{Agg}} + X_{R_{Agg}}\right)^2 \cdot \left(real\left\{Z_{i_{eq}}\right\} - R_{S\,Agg}\right)$$

$$\beta = -R_{R_{Agg}} \cdot X_{M_{Agg}}^2$$

$$\gamma = \left(real\left\{Z_{i_{eq}}\right\} - R_{S\,Agg}\right) \cdot R_{R_{Agg}}^2$$

Then, we have

$$s_{Agg} = \frac{-\beta \pm \sqrt{\beta^2 - 4 \cdot \alpha \cdot \gamma}}{2 \cdot \alpha} \tag{19}$$

It should be noted that even though Eq. (18) has the same formulation as Eq. (26) in [9], this work simplifies the calculations of α, β, and γ, because Eqs. (1) to (17) demonstrate a simplified model.

As commonly assumed, the mechanical output power from the aggregate motor is equal to the total mechanical output power from all individual induction motors. This is given by

$$P_{Agg} = \sum_{i=1}^{N} P_i \qquad , i=1,2,\ldots,N \tag{20}$$

The moment of inertia is obtained from the following relation [7],

$$J_{Agg} = \sum_{i=1}^{N} J_i \cdot \left(\frac{\omega_{r_i}}{\omega_{r_{Agg}}}\right)^2 \tag{21}$$

where ω_{r_i} and $\omega_{r_{Agg}}$ are the rotor angular speeds of the i^{th} individual motor and the aggregate motor, respectively.

III. SIMULATION RESULTS

The current work uses MATLAB-based software package including Simulink and SimPowerSystems for simulations. Five induction motors connected to the same bus are considered for a case study. All parameters and data of the induction motors [7] are shown in Table 1. The 3-phase bus voltage is 460 V for all induction motors. The system model used in this study for the aggregation method is shown in Figs. 5a and 5b.

The aggregate motor parameters, as shown in the right-most column in Table 1, are obtained from Eqs. (1) to (21).

Simulation results have been presented considering the aggregate induction motor model as one case and the individual induction motors connected to the same bus as another case. The results are compared to show the performance of the aggregate motor model. The electrical quantities obtained from the steady-state operating conditions of the aggregation motor model and five individual motors are shown in Table 2. The steady-state operating quantities of the aggregate motor are very similar to the case of five individual motors. This shows the effectiveness of the aggregate model in the steady state.

The results for the steady-state operation are shown in Figs. 6 to 7, listed in the Appendix for better illustration and easier comparison. Fig. 6(a) shows the active and reactive power of the aggregate induction motor, while Fig. 7(a) shows the sum of the active and reactive power of the five individual motors. The phase A current of the aggregate model and summation of the individual motors are illustrated in Fig. 6(b) and Fig. 7(b), respectively. The mechanical torque of the aggregate motor and the mechanical torque summation of the individual motors are shown in Fig. 6 (c) and Fig. 7(c), respectively. Figs. 6 and 7 demonstrate that the results from the aggregation model are very close to the exact model with five individual motors.

To verify the aggregate model for transient analysis, a switching event is simulated. In the simulation, all motors are assumed to start from still condition to the steady-state condition. Then, all motors are disconnected at 2.0 second from the supply for 8 cycles by opening a circuit breaker. The simulation results are presented in Figs. 8 and 9, also listed in the Appendix. The graphs clearly demonstrate a close resemblance of the result obtained from the aggregation model to that obtained from the individual motor models.

Table 1. Motor Parameters For Five Individual and One Aggregate Motors

	Induction Motors					
	1	2	3	4	5	Aggregate
P [HP]	3	15	30	50	100	198
R_S [Ω]	4.86	1.48	0.73	0.42	0.25	0.1183
X_S [Ω]	2.67	0.18	0.16	0.15	0.10	0.0384
R_R [Ω]	1.84	0.31	0.16	0.14	0.08	0.0345
X_R[Ω]	2.67	0.18	0.16	0.15	0.10	0.0384
X_M[Ω]	84.68	24.89	14.96	9.47	3.97	2.1025
J[kgm^2]	0.09	0.50	1.00	1.66	2.7	5.953
n_r[rpm]	1760	1765	1765	1750	1740	1749

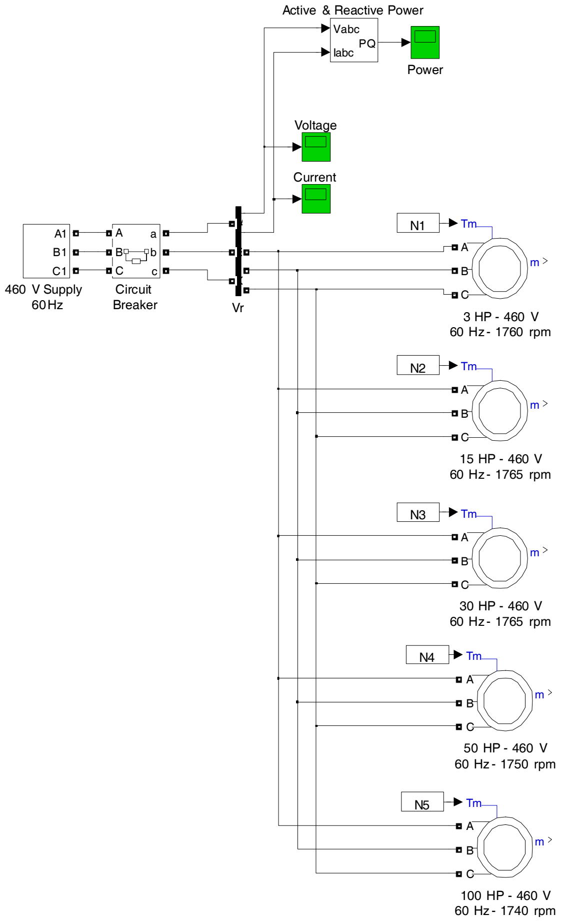

Fig. 5a. Simulation system model for individual motors.

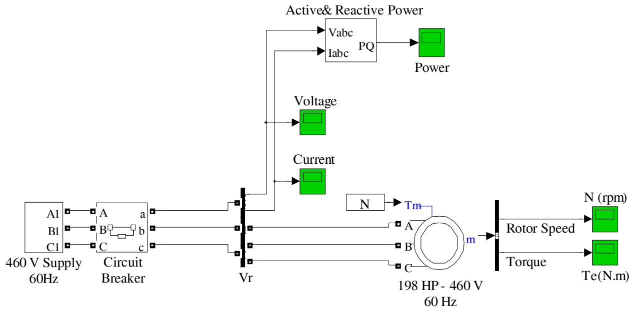

Fig. 5b. Simulation system model for aggregate motor.

Table 2. Steady-State Operating Conditions of Motors

	Sum of individual motors	Aggregate Motor
Voltage [V]	460	460
Frequency [Hz]	60	60
Stator Phase A Current [A]	227.3∠-30.42	227.1∠-30.24
Active Power [kW]	156.2	156.3
Reactive Power [kVAr]	91.69	91.11

IV. CONCLUDING REMARKS

The paper proposed an effective and effortless method to aggregate the equivalent circuit of a group of induction motors so that it can be represented by a single motor. It is assumed that the power from the aggregate motor is equal to the total power from the individual motors. The method is based on the steady-state theory of induction motors. Simulation is carried out using MATLAB software package. The results obtained from the aggregate model are compared to those obtained from the sum of the individual induction motors. The simulation results demonstrate that the performance of the aggregate motor model is satisfactory.

The model presented in this paper is simple and easy to calculate as compared to the previous work in [9]. The computation process for determining the parameters of the aggregate induction motor is neither complicated nor time-consuming. Hence, the proposed method can have a good potential to be applied in the modeling of a large number of motor loads in a complex power system.

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VI. BIOGRAPHIES

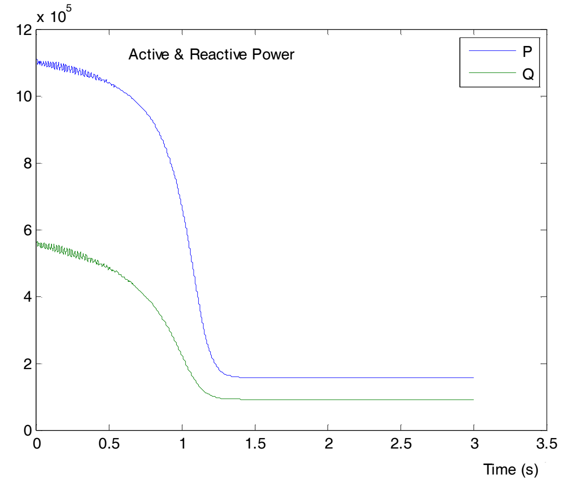
Arif Karakas received his B.S. from Istanbul Technical University in 1998, M.S. and Ph.D. degrees from Yildiz Technical University in 2000 and 2007, respectively. Dr. Karakas is presently a visiting scholar/post-doc at The University of Tennessee, Knoxville, TN. His research interests include motor modeling, voltage stability, and unit commitment.

Fangxing (Fran) Li (M'01, SM'05) received the Ph.D. degree from Virginia Tech in 2001. He has been an Assistant Professor at The University of Tennessee (UT), Knoxville, TN, USA, since August 2005. Prior to joining UT, he was a principal engineer at ABB Electrical System Consulting (ESC). During his 4 and a half years employment at ABB, he has been the lead developer of GridView^{TM}, ABB's market simulation software tool. His current interests include energy market, reactive power, distributed energy resources, distribution systems, reliability, and load modeling. He was the recipient of 2006 Eta Kappa Nu Outstanding Teacher Award at UT, voted by undergraduate ECE students. He also received UT Professional Development Award in 2007 and UT-EECS Early Career Award in 2008. Dr. Li is a registered Professional Engineer (PE) in the state of North Carolina.

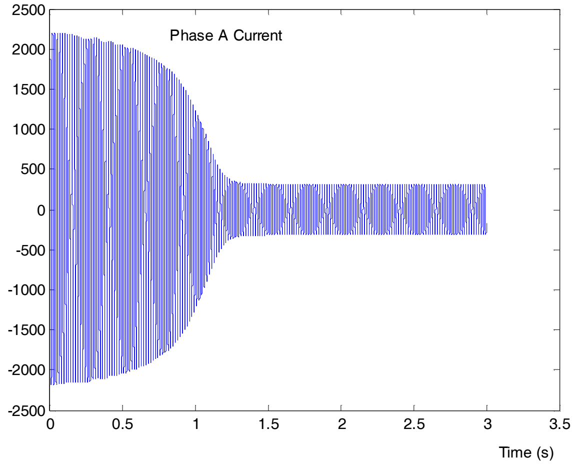
Sarina Adhikari (S'08) is presently a Ph.D. student in Electrical Engineering at The University of Tennessee. She received her B.E. degree in Electrical Engineering from Institute of Engineering, Pulchowk Campus, Pulchowk, Lalitpur, Nepal in 2002 and her M.E. degree in Electrical Power Systems Management from Asian Institute of Technology, Pathumthani, Thailand, in 2005. She worked as a Research Associate in Energy Field of Study, Asian Institute of Technology in Thailand from 2005 to 2007.

VII. APPENDIX

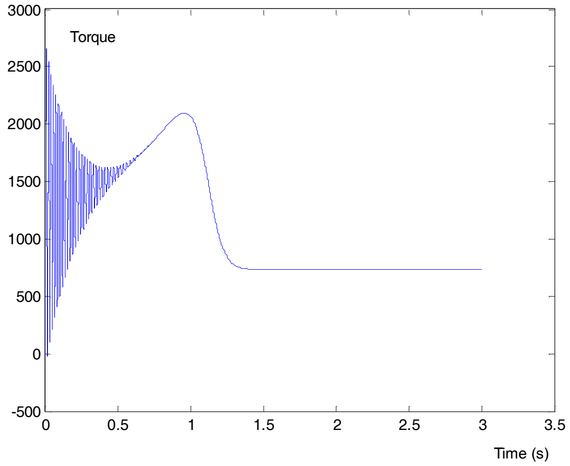
Figs. 6-9 are shown in the next two pages for better illustration of the simulation results. The curves from the aggregate model and the individual motor model are presented side by side for easy comparison and verification.



6(a): Active & reactive power responses

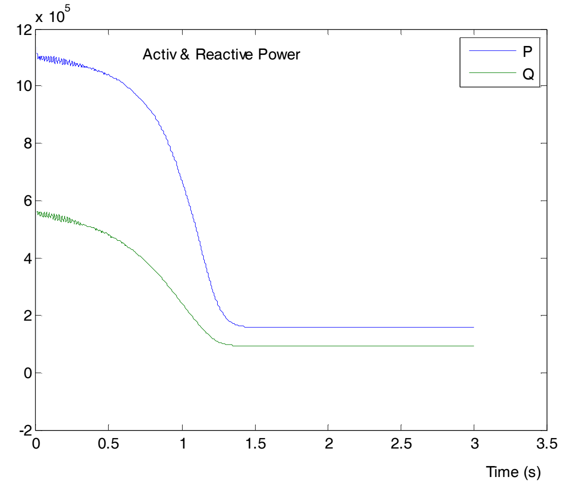


6(b): Stator phase A current

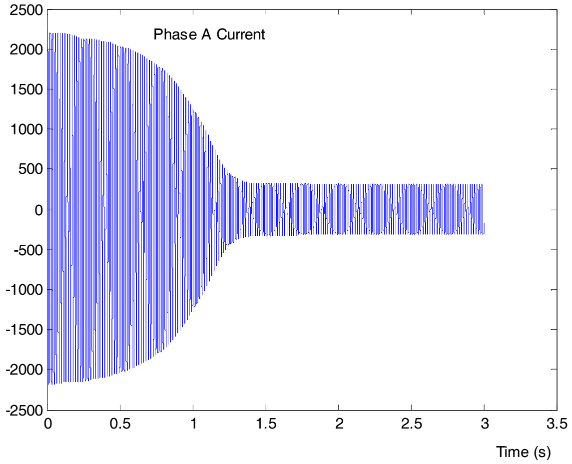


6(c): Torque

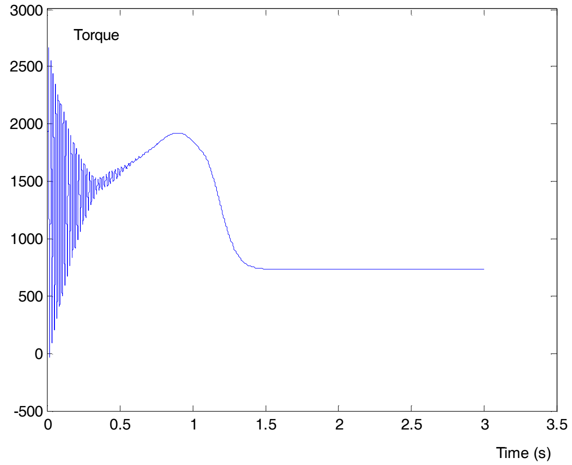
Fig. 6. Steady-state operation of the aggregate induction motor.



7(a): Summation of active & reactive power responses

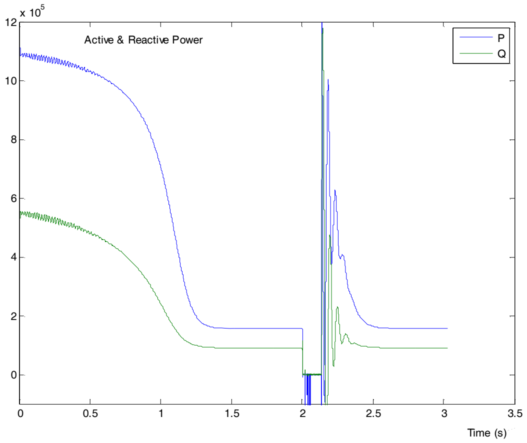


7(b): Summation of stator phase A current

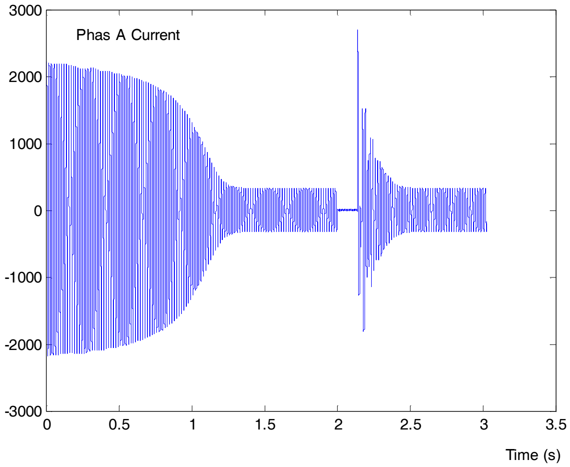


7(c): Summation of torques

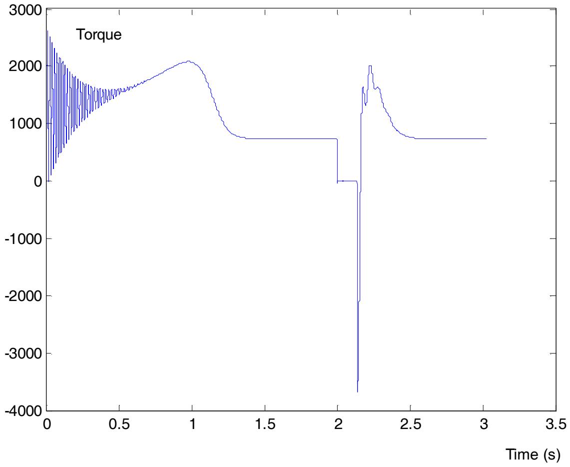
Fig. 7. Steady-state operation of individual induction motors.



8(a): Active & reactive power responses

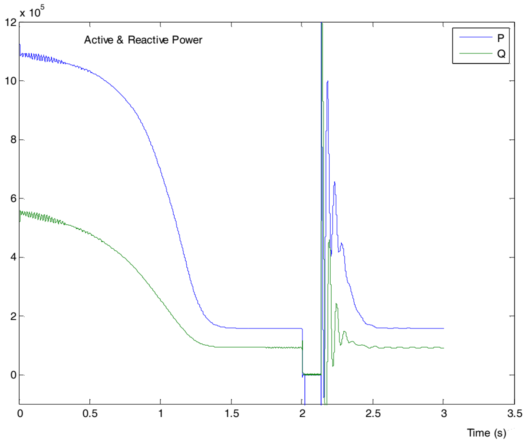


8(b): Stator phase A Current

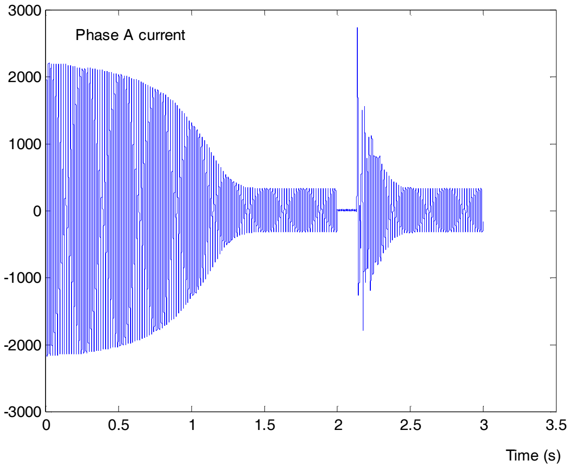


8(c): Torque

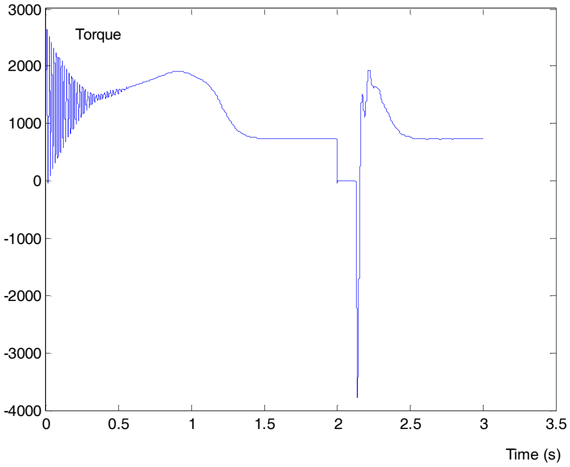
Fig. 8. Switching transient (at 2.0 second) of the aggregate induction motor model.



9(a): Summation of active & reactive power responses



9(b): Summation of stator phase A Current



9(c): Summation of torque

Fig. 9. Switching transient (at 2.0 second) of five individual induction motors.