

# A New Approach to the Elimination of Harmonics in a Multilevel Converter

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*Keywords-* multilevel converter, converter control, harmonics, drives for HEV

*Acknowledgements* Dr. Tolbert would like to thank the National Science Foundation for partially supporting this work through contract NSF ECS-0093884. Drs. Chiasson and Tolbert are grateful to Oak Ridge National Laboratory for partially supporting this work through the UT/Battelle contract no. 4000007596.

## ABSTRACT

A method is presented to compute the switching angles in a multilevel converter so as to produce the required fundamental voltage while at the same time not generate higher order harmonics. Previous work has shown that the transcendental equations characterizing the harmonic content can be converted to polynomial equations which are then solved using the method of resultants from elimination theory. However, when there are several DC sources, the degrees of the polynomials are quite large making the computational burden of their resultant polynomials via elimination theory quite high. Here, it is shown that by reformulating the problem in terms of power sums, the degrees of the polynomial equations that must be solved are reduced significantly which in turn reduces the computational burden. In contrast to numerical techniques, the approach here produces all possible solutions.

## 1 Introduction

A multilevel inverter is a power electronic device built to synthesize a desired ac voltage from several levels of dc voltages. A key issue in the design of such an inverter is to determine the switching angles (times) so as to produce the fundamental voltage and not generate specific higher order harmonics. In this work, techniques are given that allow one to control a multilevel inverter in such a way that it is an efficient, low total harmonic distortion (THD) inverter that can be used to interface distributed dc energy sources to a main ac grid.

Previous work in [1][2][3] has shown that the transcendental equations characterizing the harmonic content can be converted into *polynomial equations* which are then solved using the method of *resultants* from *elimination theory* [5][8]. However, if there are several dc sources, the degrees of the polynomials in these equations are large. As a result, one reaches the limitations of the capability of contemporary computer algebra software tools (e.g., MATHEMATICA or MAPLE) to solve the system of polynomial equations using elimination theory.

A major distinction between the work in [1][2][3] and the work in this paper is that here the theory of *power sums* [5] is exploited to reduce the degree of the polynomial equations that must be

solved so that they are well within the capability of existing computer algebra software tools. As in [3], the approach presented in this work produces all possible solutions in contrast to numerical techniques. Experimental verification that the low order harmonics are indeed eliminated is also presented by driving a three-phase induction motor from an 11-level inverter.

## 2 Cascade Multilevel Inverter

A cascade multilevel inverter consists of a series of H-bridge (single-phase full-bridge) inverter units. The inverter synthesizes a desired voltage from several separate dc sources (SDCSs), which may be obtained from solar cells, fuel cells, batteries, ultracapacitors, etc. The left side of Figure 1 shows a single-phase structure of a cascade inverter with SDCSs [9].

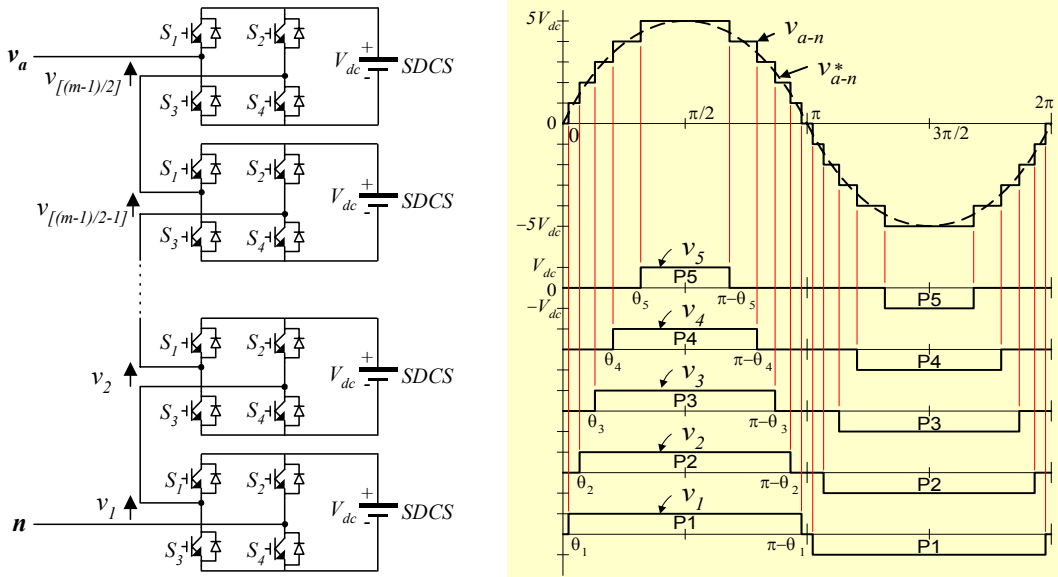


Figure 1: Left: Single-phase structure of a multilevel cascaded H-bridges inverter. Right: Output waveform of an 11-level cascade multilevel inverter.

Each SDCS is connected to a single-phase full-bridge inverter. Each inverter level can generate three different voltage outputs,  $+V_{dc}$ ,  $0$  and  $-V_{dc}$  by connecting the dc source to the ac output side by different combinations of the four switches,  $S_1, S_2, S_3$  and  $S_4$ .

The ac output of each level's full-bridge inverter is connected in series such that the synthesized voltage waveform is the sum of all of the individual inverter outputs. The number of output phase voltage levels in a cascade multilevel inverter is then  $2s + 1$ , where  $s$  is the number of dc sources. An example phase voltage waveform for an 11-level cascaded multilevel inverter with five SDCSs ( $s = 5$ ) and five full bridges is shown on the right side of Figure 1.

The output phase voltage is given by  $v_{an} = v_{a1} + v_{a2} + v_{a3} + v_{a4} + v_{a5}$ . With enough levels and an *appropriate* switching algorithm, the multilevel inverter results in an output voltage that is almost sinusoidal. For the 11 - level example shown in Figure 1, the waveform has less than 5% total harmonic distortion (THD) with each of the H-bridges' active devices switching only at the fundamental frequency.

### 3 Mathematical Model of Switching for the Multilevel Converter

Following the development in [3] (see also [12][14][15]), the Fourier series expansion of the (staircase) output voltage waveform of the multilevel inverter as shown in Figure 1 is

$$V(\omega t) = \frac{4V_{dc}}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \left( \cos(n\theta_1) + \cos(n\theta_2) + \dots + \cos(n\theta_s) \right) \sin(n\omega t) \quad (1)$$

where  $s$  is the number of dc sources. Ideally, given a desired fundamental voltage  $V_1$ , one wants to determine the switching angles  $\theta_1, \dots, \theta_s$  so that (1) becomes  $V(\omega t) = V_1 \sin(\omega t)$ . In practice, one is left with trying to do this approximately. The goal here is to choose the switching angles  $0 \leq \theta_1 < \theta_2 < \dots < \theta_s \leq \pi/2$  so as to make the first harmonic equal to the desired fundamental voltage  $V_1$  and specific higher harmonics of  $V(\omega t)$  equal to zero.

For three-phase systems, the triplen harmonics in each phase need not be canceled as they automatically cancel in the line-to-line voltages. Specifically, in the case of  $s = 5$  dc sources, the desire is to cancel the  $5^{th}, 7^{th}, 11^{th}, 13^{th}$  order harmonics as they dominate the total harmonic distortion. The mathematical statement of these conditions is then

$$\begin{aligned} \frac{4V_{dc}}{\pi} (\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) + \cos(\theta_5)) &= V_1 \\ \cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) + \cos(5\theta_4) + \cos(5\theta_5) &= 0 \\ \cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) + \cos(7\theta_4) + \cos(7\theta_5) &= 0 \\ \cos(11\theta_1) + \cos(11\theta_2) + \cos(11\theta_3) + \cos(11\theta_4) + \cos(11\theta_5) &= 0 \\ \cos(13\theta_1) + \cos(13\theta_2) + \cos(13\theta_3) + \cos(13\theta_4) + \cos(13\theta_5) &= 0. \end{aligned} \quad (2)$$

This is a system of five transcendental equations in the five unknowns  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ . The question here is ‘‘When does the set of equations (2) have a solution?’’. The correct solution to the conditions (2) would mean that the output voltage of the 11–level inverter would not contain the  $5^{th}, 7^{th}, 11^{th}$ , and  $13^{th}$  order harmonic components. One approach to solving this set of nonlinear transcendental equations (2) is to use an iterative method such as the Newton-Raphson method [6][12][14][15]. In contrast to iterative methods, the following presents a new approach that obtains all possible solutions and requires significantly less computational effort than the approach in [3]. To proceed with the new methodology, first let  $s = 5$ , and define  $x_i = \cos(\theta_i)$  for  $i = 1, \dots, 5$ . Using the trigonometric identities  $\cos(5\theta) = 5\cos(\theta) - 20\cos^3(\theta) + 16\cos^5(\theta)$ ,  $\cos(7\theta) = \dots$ , the conditions (2) become

$$\begin{aligned} p_1(x) &\triangleq x_1 + x_2 + x_3 + x_4 + x_5 - m = 0 \\ p_5(x) &\triangleq \sum_{i=1}^5 \left( 5x_i - 20x_i^3 + 16x_i^5 \right) = 0 \\ p_7(x) &\triangleq \sum_{i=1}^5 \left( -7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7 \right) = 0 \\ p_{11}(x) &\triangleq \sum_{i=1}^5 \left( -11x_i + 220x_i^3 - 1232x_i^5 + 2816x_i^7 - 2816x_i^9 + 1024x_i^{11} \right) = 0 \\ p_{13}(x) &\triangleq \sum_{i=1}^5 \left( 13x_i - 364x_i^3 + 2912x_i^5 - 9984x_i^7 + 16640x_i^9 - 13312x_i^{11} + 4096x_i^{13} \right) = 0 \end{aligned} \quad (3)$$

where  $x = (x_1, x_2, x_3, x_4, x_5)$  and  $m \triangleq V_1 / (4V_{dc}/\pi)$ . The modulation index is  $m_a = m/s = V_1 / (s4V_{dc}/\pi)$ . (Each inverter has a dc source of  $V_{dc}$  so that the maximum output voltage of the

multilevel inverter is  $sV_{dc}$ . A square wave of amplitude  $sV_{dc}$  results in the maximum fundamental output possible of  $V_{1\max} = 4sV_{dc}/\pi$  so  $m_a \triangleq V_1/V_{1\max} = V_1/(s4V_{dc}/\pi) = m/s$ .

This is a set of five equations in the five unknowns  $x_1, x_2, x_3, x_4, x_5$ . Further, the solutions must satisfy  $0 \leq x_5 < x_4 < x_3 < x_2 < x_1 \leq 1$ . This development has resulted in a set of polynomial equations rather than trigonometric equations. The degrees of the polynomials are large which in turn requires the *symbolic* computation of the determinant of large square matrices. Contemporary computer algebra software tools cannot solve these equations on a personal computer for inverters with more than four dc sources [3]. Here a new approach to solving the system (3) is presented which greatly reduces the computational burden. This is done by taking into account the symmetry of the polynomials making up the system (3). Specifically, the *theory of power sums* [5] is exploited to obtain a new set of relatively low degree polynomials whose resultants can easily be computed using existing computer algebra software tools. In [11] a polynomial approach was also considered and iterative numerical techniques were used to solve the equations. However, in contrast to such numerical techniques, the approach here produces *all* possible solutions.

## 4 Solving Polynomial Equations

For the purpose of exposition, the three source (7 level) multilevel inverter will be used to illustrate the approach. The conditions are then

$$\begin{aligned} p_1(x) &\triangleq x_1 + x_2 + x_3 - m = 0, & m &\triangleq \frac{V_1}{4V_{dc}/\pi} \\ p_5(x) &\triangleq \sum_{i=1}^3 (5x_i - 20x_i^3 + 16x_i^5) = 0 \\ p_7(x) &\triangleq \sum_{i=1}^3 (-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7) = 0. \end{aligned} \tag{4}$$

Eliminating  $x_3$  by substituting  $x_3 = m - (x_1 + x_2)$  into  $p_5, p_7$  gives

$$\begin{aligned} p_5(x_1, x_2) &= 5x_1 - 20x_1^3 + 16x_1^5 + 5x_2 - 20x_2^3 + 16x_2^5 + 5(m - x_1 - x_2) - 20(m - x_1 - x_2)^3 \\ &\quad + 16(m - x_1 - x_2)^5 \\ p_7(x_1, x_2) &= -7x_1 + 56x_1^3 - 112x_1^5 + 64x_1^7 - 7x_2 + 56x_2^3 - 112x_2^5 + 64x_2^7 - 7(m - x_1 - x_2) \\ &\quad + 56(m - x_1 - x_2)^3 - 112(m - x_1 - x_2)^5 + 64(m - x_1 - x_2)^7 \end{aligned} \tag{5}$$

where  $\deg_{x_1} \{p_5(x_1, x_2)\} = 4$ ,  $\deg_{x_2} \{p_5(x_1, x_2)\} = 4$  and  $\deg_{x_1} \{p_7(x_1, x_2)\} = 6$ ,  $\deg_{x_2} \{p_7(x_1, x_2)\} = 6$ .

### 4.1 Elimination Using Resultants

In order to explain the computational issues with finding the zero sets of polynomial systems, a brief discussion of the procedure to solve such systems is now given. The question at hand is "Given two polynomial equations  $a(x_1, x_2) = 0$  and  $b(x_1, x_2) = 0$ , how does one solve them simultaneously to eliminate (say)  $x_2$ ?". A systematic procedure to do this is known as *elimination theory* and uses the notion of *resultants* [5][8]. Briefly, one considers  $a(x_1, x_2)$  and  $b(x_1, x_2)$  as polynomials in  $x_2$  whose coefficients are polynomials in  $x_1$ . Then, for example, letting  $a(x_1, x_2)$  and  $b(x_1, x_2)$  have degrees 3 and 2, respectively in  $x_2$ , they may be written in the form

$$\begin{aligned} a(x_1, x_2) &= a_3(x_1)x_2^3 + a_2(x_1)x_2^2 + a_1(x_1)x_2 + a_0(x_1) \\ b(x_1, x_2) &= b_2(x_1)x_2^2 + b_1(x_1)x_2 + b_0(x_1). \end{aligned}$$

The  $n \times n$  *Sylvester* matrix, where  $n = \deg_{x_2} \{a(x_1, x_2)\} + \deg_{x_2} \{b(x_1, x_2)\} = 3 + 2 = 5$ , is defined by

$$S_{a,b}(x_1) = \begin{bmatrix} a_0(x_1) & 0 & b_0(x_1) & 0 & 0 \\ a_1(x_1) & a_0(x_1) & b_1(x_1) & b_0(x_1) & 0 \\ a_2(x_1) & a_1(x_1) & b_2(x_1) & b_1(x_1) & b_0(x_1) \\ a_3(x_1) & a_2(x_1) & 0 & b_2(x_1) & b_1(x_1) \\ 0 & a_3(x_1) & 0 & 0 & b_2(x_1) \end{bmatrix}.$$

The *resultant* polynomial is then defined by

$$r(x_1) = \text{Res} \left( a(x_1, x_2), b(x_1, x_2), x_2 \right) \triangleq \det S_{a,b}(x_1) \quad (6)$$

and is the result of solving  $a(x_1, x_2) = 0$  and  $b(x_1, x_2) = 0$  simultaneously for  $x_1$ , i.e., eliminating  $x_2$ . The point here is that as the degrees of the polynomials increase, the size of the corresponding Sylvester matrix increases, and therefore the symbolic computation of its determinant becomes much more computationally intensive.

## 4.2 Power Sums

Consider once again the system of polynomial equations (5). In [3] (see also [1][2]) the authors computed the resultant polynomial of the pair  $\{p_5(x_1, x_2), p_7(x_1, x_2)\}$  to obtain the solutions to (4). This involved setting up a  $10 \times 10$  Sylvester matrix ( $10 = \deg_{x_2} \{p_5(x_1, x_2)\} + \deg_{x_2} \{p_7(x_1, x_2)\}$ ) and then computing its determinant to obtain the resultant polynomial  $r(x_1)$  whose degree was 22. However, as one adds more dc sources to the multilevel inverter, the degrees of the polynomials go up rapidly. For example, in the case of four dc sources, the final step of the method requires computing (symbolically) the determinant of a  $27 \times 27$  Sylvester matrix to obtain a resultant polynomial of degree 221. In the case of five sources, the authors were only able to get the system of five polynomial equations in five unknowns to reduce to three equations in three unknowns. The computation to get it down to two equations in two unknowns requires the symbolic computation of the determinant of a  $33 \times 33$  Sylvester matrix. To get around this difficulty, a new approach is developed here which exploits the fact that the polynomials in (3) are *symmetric*, i.e.,  $p_1(x_1, x_2, x_3) = p_1(x_2, x_1, x_3)$ , etc. As a result (see [5]), the polynomials  $p_1(x), p_2(x), p_3(x)$  in (4) may be written in terms of the *power sums*  $t_1, t_2, t_3$  defined as

$$t_1 \triangleq x_1 + x_2 + x_3, t_2 \triangleq x_1^2 + x_2^2 + x_3^2, t_3 \triangleq x_1^3 + x_2^3 + x_3^3, \dots \quad (7)$$

Using the power sums, the polynomials (4) become

$$\begin{aligned} p_1(t) &= t_1 - m \\ p_5(t) &= 5t_1 - 20t_3 + 16t_5 \\ p_7(t) &= -7t_1 + 56t_3 - 112t_5 + 64t_7. \end{aligned} \quad (8)$$

This is now a set of three equations in the four unknowns  $t_1, t_3, t_5, t_7$ . However the polynomials of (4) are symmetric in the  $x_i$ , i.e., for example, if one interchanges  $x_1$  and  $x_3$ , the polynomials remain the same. (This also is seen from the fact that the system (4) has been rewritten in (8) in terms of the power sums which are symmetric in the  $x_i$ .) As a result, the theory of power sums says that any set of *symmetric polynomials* in the variables  $x_1, x_2, \dots, x_n$  can be rewritten in terms of the power sums  $t_1, t_2, \dots, t_n$  (see [5] page 317). In the case of (8), this means that  $t_5, t_7$  can be written as polynomials in  $t_1, t_2, t_3$ . Specifically,

$$\begin{aligned} t_5 &= \left( t_1^5 - 5t_1^3t_2 + 5t_1^2t_3 + 5t_2t_3 \right) / 6 \\ t_7 &= \left( t_1^7 - 21t_1^3t_2^2 + 7t_1^4t_3 + 21t_2^2t_3 + 28t_1t_3^2 \right) / 36. \end{aligned}$$

These expressions are then substituted into (8) to get

$$\begin{aligned} p_1(t) &= t_1 - m \\ p_5(t) &= \left(15t_1 + 8t_1^5 - 40t_1^3 + t_2 - 60t_3 + 40t_1^2t_3 + 40t_2t_3\right) / 3 \\ p_7(t) &= \left(-63t_1 - 168t_1^5 + 16t_1^7 + 840t_1^3t_2 - 336t_1^3t_2^2 + 504t_3 - 840t_1^2t_3 + 112t_1^4t_3 - 840t_2t_3 \right. \\ &\quad \left. + 336t_2^2t_3 + 448t_1t_3^2\right) / 9 \end{aligned}$$

where  $t \triangleq (t_1, t_2, t_3)$ . This is now a system of three polynomials in three unknowns. One uses  $p_1(t) = t_1 - m = 0$  to eliminate  $t_1$  so that

$$\begin{aligned} q_5(t_2, t_3) \triangleq p_5(m, t_2, t_3) &= \left(15m + 8m^5 - 40m^3 + t_2 - 60t_3 + 40m^2t_3 + 40t_2t_3\right) / 3 \\ q_7(t_2, t_3) \triangleq p_7(m, t_2, t_3) &= \left(-63m - 168m^5 + 16m^7 + 840m^3t_2 - 336m^3t_2^2 + 504t_3 - 840m^2t_3 + 112m^4t_3 \right. \\ &\quad \left. - 840t_2t_3 + 336t_2^2t_3 + 448mt_3^2\right) / 9 \end{aligned}$$

where  $\deg_{t_2} \{q_5(t_2, t_3)\} = 1$ ,  $\deg_{t_3} \{q_5(t_2, t_3)\} = 1$  and  $\deg_{t_2} \{q_7(t_2, t_3)\} = 2$ ,  $\deg_{t_3} \{q_7(t_2, t_3)\} = 2$ . The key point here is that degrees of these polynomials in  $t_2, t_3$  are much less than the degrees of  $p_5(x_1, x_2), p_7(x_1, x_2)$  in  $x_1, x_2$  (see equation (5)). In particular, the Sylvester matrix of the pair  $\{q_5(t_2, t_3), q_7(t_2, t_3)\}$  is  $3 \times 3$  rather than being  $10 \times 10$  in the case of  $\{p_5(x_1, x_2), p_7(x_1, x_2)\}$  in (5). Eliminating  $t_2$ , the resultant polynomial  $\text{Res}(q_5(t_2, t_3), q_7(t_2, t_3), t_2)$  is given by

$$\begin{aligned} \text{Res}(q_5(t_2, t_3), q_7(t_2, t_3), t_2) &= \frac{16}{81}m(m^3 - t_3) \left(-4725m + 25200m^3 - 5040m^5 + 256m^9 + 12600t_3 \right. \\ &\quad \left. - 100800m^2t_3 + 20160m^4t_3 - 3840m^6t_3 + 100800t_2t_3^3 - 44800t_3^3\right) \end{aligned}$$

which factors into a polynomial of degree 1 in  $t_3$  and of degree 3 in  $t_3$ . For each  $m$ , one solves  $\text{Res}(q_5, q_7, t_2) = 0$  for the roots  $\{t_{3i}\}_{i=1,2,3}$ . These roots are then used to solve  $q_5(t_2, t_{3i}) = 0$  for the root  $t_{2i}$  resulting in the set of 3-tuples  $\{(t_1, t_2, t_3) \in \mathbb{C}^3 \mid (t_1, t_2, t_3) = (m, t_{2i}, t_{3i}) \text{ for } i = 1, 2, 3\}$  as the only possible solutions to (8).

### 4.3 Solving the Power Sums

For each solution triple  $(t_1, t_2, t_3)$ , the corresponding values of  $(x_1, x_2, x_3)$  are required to obtain the switching angles. To do so, one simply uses the resultant method to solve the system of polynomials

$$\begin{aligned} f_1(x_1, x_2, x_3) &= t_1 - (x_1 + x_2 + x_3) = 0 \\ f_2(x_1, x_2, x_3) &= t_2 - (x_1^2 + x_2^2 + x_3^2) = 0 \\ f_3(x_1, x_2, x_3) &= t_3 - (x_1^3 + x_2^3 + x_3^3) = 0. \end{aligned}$$

for  $(x_1, x_2, x_3)$ . That is, one computes

$$\begin{aligned} r_1(x_2, x_3) &\triangleq \text{Res}\left(f_1(x_1, x_2, x_3), f_2(x_1, x_2, x_3), x_1\right) = t_1^2 - t_2 - 2t_1x_2 + 2x_2^2 - 2t_1x_3 + 2x_2x_3 + 2x_3^2 \\ r_2(x_2, x_3) &\triangleq \text{Res}\left(f_1(x_1, x_2, x_3), f_3(x_1, x_2, x_3), x_1\right) \\ &= t_1^3 - t_3 - 3t_1^2x_2 + 3t_1x_2^2 - 3t_1^2x_3 + 6t_1x_2x_3 - 3x_2^2x_3 + 3t_1x_3^2 - 3x_2x_3^2 \end{aligned}$$

and finally

$$r(x_3) \triangleq \text{Res}\left(r_1(x_2, x_3), r_2(x_2, x_3), x_2\right) = \left(t_1^3 - 3t_1t_2 + 2t_3 - 3t_1^2x_3 + 3t_2x_3 + 6t_1x_3^2 - 6x_3^3\right)^2. \quad (9)$$

The procedure is to substitute the solutions of (8) into (9) and solve for the roots  $\{x_{3i}\}$ . For each  $x_{3i}$ , one then solves  $r_1(x_2, x_{3i})$  for the roots  $x_{2j}$ . Finally, one solves  $f_1(x_1, x_{2j}, x_{3i}) = 0$  for  $x_{1j}$  to get the triples  $\{(x_1, x_2, x_3) = (x_{1j}, x_{2j}, x_{3i}), i = 1, 2, 3, j = 1, 2\}$  as the only possible solutions to (4). This finite set of possible solutions can then be checked as to which are solutions of (4) satisfying  $0 \leq x_3 < x_2 < x_1 \leq 1$ .

## 5 Computational Results

For the case of five DC sources and using the fundamental switching scheme of Figure 1, the complete set of solutions to (2) were computed using the method described in the previous section. These solutions are plotted on the left side of Figure 2 versus the parameter  $m$ . As the plots show, for  $m$  in the intervals  $[1.88, 1.89]$ ,  $[2.21, 3.66]$  and  $[3.74, 4.23]$ , the output waveform can have the desired fundamental with the 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup> harmonics absent. Further, in the subinterval  $[2.53, 2.9]$  two sets of solutions exist, while in the subinterval  $[3.05, 3.29]$ , there are three sets of solutions. In the case of multiple solution sets, one would typically choose the set that gives the lowest total harmonic distortion (THD). In those intervals for which no solutions exist; one must use a different switching scheme (see [4] for a discussion on such possibilities).

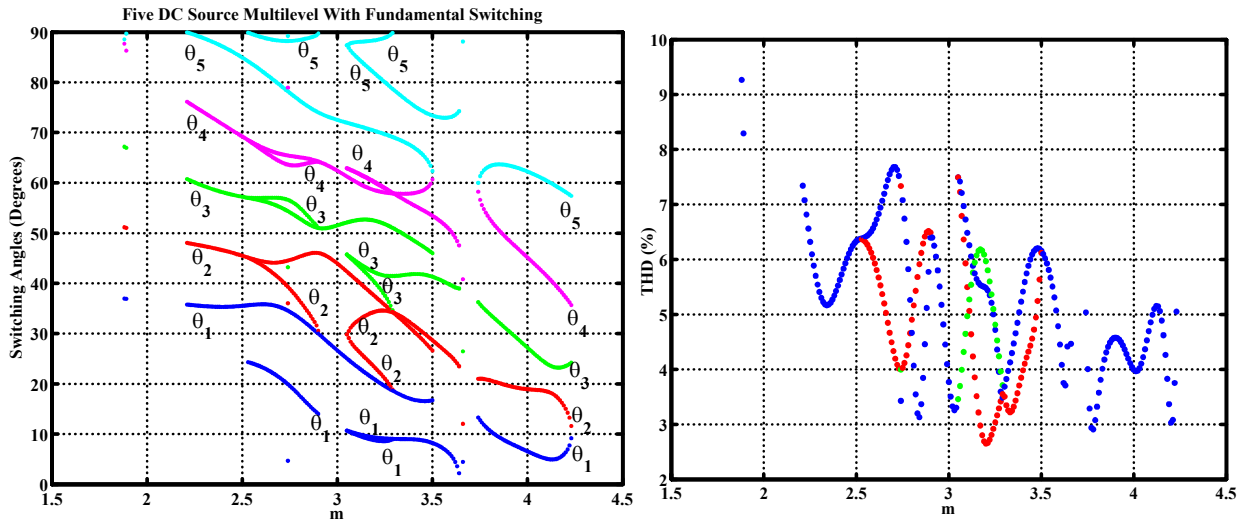


Figure 2: Left: Switching angles vs  $m$  for the 5 dc source multilevel converter. Right: THD vs  $m$  for each solution set ( $m_a = m/s$  with  $s = 5$ ).

The corresponding total harmonic distortion (THD) was computed out to the 31<sup>st</sup> according to

$$THD = \sqrt{(V_5^2 + V_7^2 + V_{11}^2 + V_{13}^2 + V_{17}^2 + \dots + V_{31}^2) / V_1^2}$$

and is plotted versus  $m$  on the right side of Figure 2 for each of the solution sets shown on the left side of this same figure. As this figure shows, one can choose a particular solution for the switching angles such that the THD is 6.5% or less for  $2.25 \leq m \leq 4.23$  ( $0.45 \leq m_a \leq 0.846$ ).

For those values of  $m$  for which multiple solution sets exist, an appropriate choice is the one that results in the lowest THD. For example, Figure 2 shows that there is a solution set for  $m$  in the interval  $[2.21, 3.66]$  that is continuous as a function of  $m$ , but it is seen that in the subintervals

[2.8, 2.9] and [3.11, 3.29], one chooses a different solution set to obtain a smaller THD. A look at Figure 2 shows that this difference in THD can be as much as 3.5% which is significant.

If one had used an iterative method such as Newton-Raphson, then only one solution set would be found, and it would most certainly not be the solution set that results in the lowest THD for  $m$  in the subintervals [2.8, 2.9] and [3.11, 3.29]. The reason the Newton-Raphson method would not have found this solution set is simply due to the way it is implemented. One starts with an initial guess for the angles at  $m = 2.21$  (It would take some guessing to even know what value of  $m$  to start with!). Then the solution set for this value of  $m$  would be used as the initial guess for the solution when  $m$  is incremented by  $\Delta m$  to its next value and so on. At  $m = 2.21$ , there is only one possible solution as Figure 2 shows. Then, as  $m$  is incremented, the Newton-Raphson algorithm would give the solution set in Figure 2 that is *continuous* as a function of  $m$ , which is not always the solution set with the smallest THD. In contrast, the method proposed here finds the complete solution set and allows one to be sure that the solution with the lowest THD is used.

## 6 Experimental Results

The experimental setup is a three-phase 11-level (5 dc sources) wye-connected cascaded inverter using 100 V, 70 A MOSFETs as the switching devices [13]. A battery bank of 15 SDCSs of 36 V (not shown) each feed the inverter (5 SDCSs per phase). The step size for the real time implementation was 32 microseconds. This small step was used to obtain an accurate resolution for implementing the switching times.

Note that while the computation of the data plotted in Figure 2 requires some offline computational effort, the real-time implementation is accomplished by putting this data (i.e., Figure 2) in a lookup table and therefore does not require high computational power for implementation. The multilevel converter was attached to a three phase induction motor with a rated hp of 1/3 hp, a rated current of 1.5 A, a rated speed of 1725 rpm and a rated voltage of 208 V (RMS line-to-line at 60 Hz).

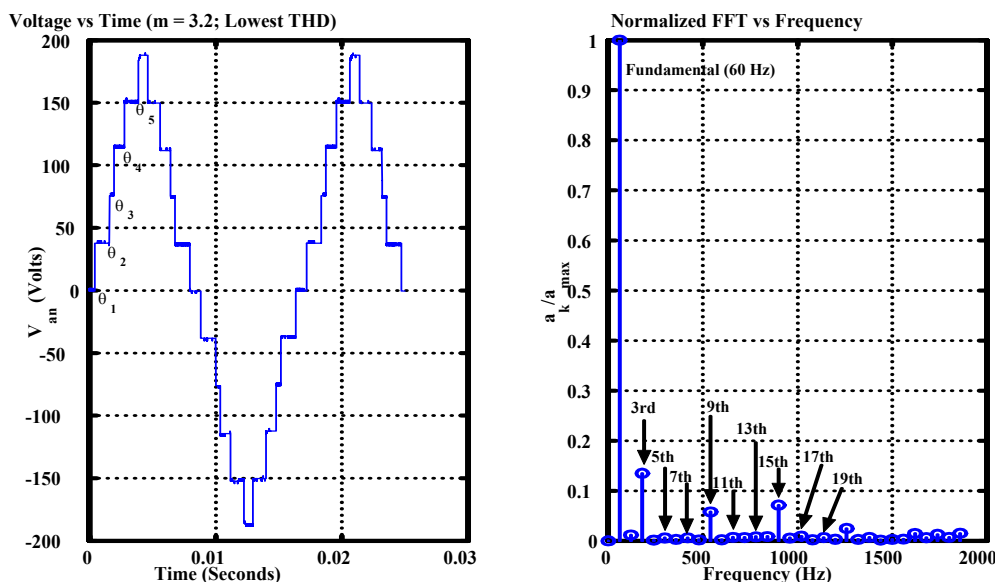


Figure 3: Phase  $a$  output voltage waveform ( $m = 3.2$ ) using the solutions set with the lowest THD and its normalized FFT.

In this experiment,  $m = 3.2$  was chosen to produce a fundamental voltage of  $V_1 = m(4V_{dc}/\pi) = 3.2(4 \times 36/\pi) = 146.7$  V along with  $f = 60$  Hz. As can be seen in Figure 2, there are three different solution sets for  $m = 3.2$ . The solution set that gave the smallest THD ( $= 2.65\%$ , see Figure 2) was used. Figure 3 shows the phase  $a$  voltage and its corresponding FFT showing that the 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup> and 13<sup>th</sup> are absent from waveform as predicted. The THD of the line-line voltage was computed using the data in Figure 3 and was found to be 2.8%, comparing favorably with the value of 2.65% predicted in Figure 2. The motor current of phase  $a$  corresponding to the output voltage of Figure 3 is shown on the left side of Figure 4. The right side is the FFT of this current whose THD was found to be 1.9% which is less than the voltage due to the filtering by the motor's inductance.

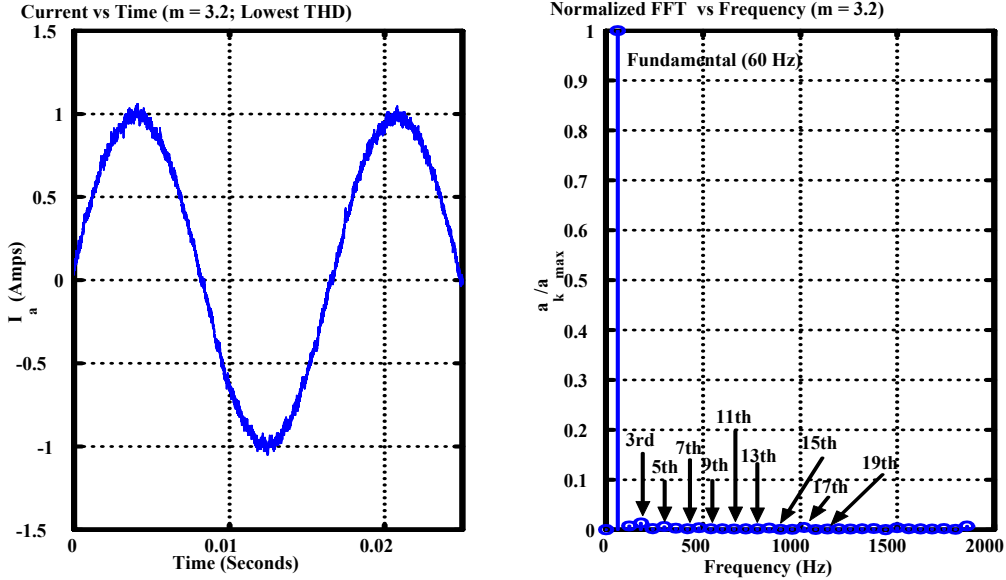


Figure 4: Phase  $a$  current corresponding to the voltage in Figure 3 and its normalized FFT.

## 7 Conclusions

A procedure to eliminate harmonics in a multilevel inverter has been given which exploits the properties of the transcendental equations that define the harmonic content of the converter output. Specifically, it was shown that one can transform the transcendental equations into symmetric polynomials which are then further transformed into another set of polynomials in terms of the elementary symmetric functions.

This formulation resulted in a drastic reduction in the degrees of the polynomials that characterize the solution. Consequently, the computation of solutions of this final set of polynomial equations were easily carried out using elimination theory (resultants) as the required symbolic computations were well within the capabilities of contemporary computer algebra software tools. This methodology resulted in the complete characterization of the solutions to the harmonic elimination problem. That is, for each  $m$ , it produces *all* possible solutions or it shows that no solution exists. This is in contrast to numerical techniques such as Newton-Raphson, optimization software, etc. (for example, see [7],[10]) where one gets only one solution or no solution and is left to ponder whether a solution exists or not. Experiments were performed and the data presented corresponded well with the predicted results.

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